Nominal GDP Targeting and the Zero Lower Bound: Should We Abandon Inflation Targeting?*

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Sveriges Riksbank Working Paper Series No. 270
Revised May 2014

Abstract

I compare nominal GDP level targeting to flexible inflation targeting in a small New Keynesian model subject to the zero lower bound on nominal policy rates. I first test the two targets under central bank discretion and find that, for a plausible calibration, inflation targeting leaves the economy vulnerable to a deflationary trap, while nominal GDP targeting provides a firm nominal anchor. Next, I test the targets under simple policy rules with forward guidance. In this case, an inflation target outperforms a nominal GDP target, because it allows the central bank to give guidance directly on interest rates, rather than indirectly on a level of GDP it seeks to achieve.

Keywords: nominal GDP target, optimal policy, simple rules, zero lower bound

JEL: E31, E52, E58

*I thank for comments seminar participants at Danmarks Nationalbank, Sveriges Riksbank and University of Glasgow Adam Smith Business School, as well as participants at the SCE meeting. The views expressed herein are solely the responsibility of the author and should not be interpreted as reflecting the views of the Executive Board of Sveriges Riksbank.

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1 Introduction

With policy interest rates near zero and a weak economy, flexible inflation targeting, as practiced by major central banks, has come under intense public scrutiny and criticism. A conceptually appealing alternative, some argue, would be to adopt nominal GDP level targeting. In this case, the central bank would have to make up for any past deviations of nominal GDP from the target. In particular, when nominal GDP falls below target, the central bank has to compensate for the shortfall in economic activity by credibly promising further policy stimulus. Such forward policy guidance on the level of nominal GDP provides a way to stimulate the economy when policy is constrained by the zero lower bound (ZLB) on nominal interest rates. Thus, according to the relevant literature, nominal GDP level targeting is clearly a superior framework for monetary policy compared to inflation targeting.

In this article, however, I show that flexible inflation targeting is actually superior to nominal GDP level targeting as a monetary policy framework. Specifically, I study the two targets in a small New Keynesian model subject to the ZLB constraint. The stylized model allows for a clear intuition about the key tradeoffs faced by policymakers under the two targets. I begin by testing the targets under optimal central bank discretion. I find that, for a plausible calibration of the model to U.S. data, inflation targeting leaves the economy vulnerable to a calamitous deflationary trap from which there is no escape. If the inflation target is below a critical value, a bad shock can push the economy into a deflationary spiral, with rising rates of deflation sending real interest rates soaring and the economy into a tailspin. By contrast, nominal GDP level targeting provides a firm nominal anchor to the economy.

Next, I test the two targets under simple policy rules with forward guidance. In this case, forward guidance involves adjustments to the setting of the central bank’s policy interest rate, to influence the expectations of the future path of real interest rates. I find that, an inflation target performs better than a nominal GDP target, because it allows the central bank to give guidance directly on interest rates, rather than indirectly on a level of GDP it seeks to achieve. With forward guidance on interest rates, inflation is more tightly anchored and therefore the economy is less vulnerable to a deflationary spiral. Furthermore, the greater the uncertainty
about the evolution of the economy, the worse the performance of a nominal GDP target. Thus, the analysis suggests that, contrary to some recent proposals, inflation targeting should clearly not be ditched.

In the New Keynesian literature, the case for nominal GDP level targeting has recently been stressed by Woodford (2012). In fact, nominal GDP level targeting resembles the ideal policy, i.e., the Ramsey plan, in the small New Keynesian model as studied in Woodford (2010). The desirability of a price level target when the ZLB becomes a binding constraint was stressed by Eggertsson and Woodford (2003) and Svensson (2003), well before the financial crisis and great recession. More recently in the aftermath of the crisis, Billi (2011) and Coibion, Gorodnichenko, and Wieland (2012) study the optimal rate of inflation and the ZLB in the New Keynesian model. While Coibion, Gorodnichenko, and Wieland (2012) argue that price level targeting makes the exit from a ZLB episode occur more rapidly, this article shows that it makes the recession deeper and economic recovery slower than can be achieved otherwise. With forward guidance on interest rates, inflation is more firmly anchored in the economy.

As in Billi (2011), I use a New Keynesian model linearized around zero inflation and account for the costs of steady-state inflation in the welfare calculation directly. In this article, I find that the optimal inflation rate is below 2 percent annual under optimal central bank discretion and even falls below 0.2 percent annual under simple policy rules with forward guidance. This low level of trend inflation helps to ensure that the welfare ranking of the alternative policies is essentially correct. In fact, Coibion, Gorodnichenko, and Wieland (2012) incorporate into a linearized New Keynesian model the effects of positive trend inflation, along the lines of Ascari and Ropele (2009), but find that the welfare costs of inflation (p. 17) “are essentially driven by only two components: the steady-state effect and the contribution of inflation variance to utility.”1 Thus, in practice, the two modelling approaches are equally effective in capturing the overall effects from positive trend inflation.2 Still, those works do not study the anchoring

1More specifically, Coibion, Gorodnichenko, and Wieland (2012) linearize the model and perform welfare approximations under the assumption of positive trend inflation. Conceptually, their approach captures also two other effects of positive trend inflation, i.e., on the weight assigned to inflation in utility and on the dynamics of the model. But, in practice, accounting for those two effects does not lead to a noticeable change in utility as a function of trend inflation, as figures 2B and 3 in their paper shows.

2There is, however, a difference in the numerical approach, which, in turn, has implications for the expected
effect of forward guidance, as done instead in this article.

The analysis is conducted in an arguably stylized model, which does not include capital nor accounts for costs associated with a change in the policy framework. As noted, for example, by Coibion, Gorodnichenko, and Wieland (2012), the introduction of capital in the analysis affects the costs of inflation and the likelihood of a ZLB episode, because capital permits disinvestment in a ZLB episode. In such a setting, however, the central bank still has the incentive to give guidance directly on interest rates, rather than indirectly on a level of GDP it seeks to achieve, to counter downward pressure on inflation in a ZLB episode. Nevertheless, one could argue that central banks should adopt nominal GDP level targeting, and thus use forward guidance both on nominal GDP and policy rates. Of course, this line of reasoning implies a fundamental change in the current monetary policy framework of major central banks. But the analysis does not take into account any potential costs associated with a change in the policy framework, such as the uncertainty faced by households and firms about the occurrence of a regime change. Accounting for the costs of regime change, the central bank would have an even stronger incentive to give guidance directly on interest rates, to anchor inflation firmly in the economy.

Section 2 describes the model. Section 3 presents the policy evaluation. And Section 4 concludes. The Appendix contains technical details.

2 The model

I use a small New Keynesian model as described in Woodford (2010), but with a nominal GDP level target hardwired into the central bank’s objective function. I also introduce the target in a class of simple policy rules studied by Taylor and Williams (2010). In addition, I take into account that the nominal policy rate occasionally hits the ZLB. In such a setting, I explain how to account for the steady-state costs of inflation in the policy evaluation. After
describing the salient features of an equilibrium that accounts for the ZLB and uncertainty about the evolution of the economy, I calibrate the model to U.S. data.

2.1 Private sector

The behavior of the private sector is summarized by two log-linearized, structural equations, namely an Euler equation and a Phillips curve, respectively describing the demand and supply side of the economy. The equations of this basic model are linearized around zero inflation.

The Euler equation, which describes the representative household’s expenditure decisions, is given by

\[ x_t = E_t x_{t+1} - \varphi (i_t - E_t \pi_{t+1} - r^n_t), \tag{1} \]

where \( E_t \) denotes the expectations operator conditional on information available at time \( t \). \( x_t \) denotes the measure of real economic activity or the real GDP gap, i.e., the deviation of real GDP from its flexible-price equilibrium. \( \pi_t \) is the inflation rate, i.e., the change in the log-price level (\( \pi_t = p_t - p_{t-1} \)). \( i_t \geq 0 \) denotes the short-term nominal interest rate (as well as the instrument of monetary policy, as discussed in the next subsection). And \( r^n_t \) is a natural rate of interest shock.\(^3\) \( \varphi > 0 \) is the interest elasticity of real aggregate demand, capturing intertemporal substitution in household’s spending.

The Phillips curve, which describes the optimal price-setting behavior of firms, under staggered price changes à la Calvo, is given by

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t, \tag{2} \]

where \( u_t \) is a mark-up shock, resulting from variation over time in the degree of monopolistic competition between firms. \( \beta \in (0,1) \) denotes the discount factor of the representative household.

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\(^3\)The shock \( r^n_t \) summarizes all shocks that under flexible prices generate variation in the real interest rate. It captures the combined effects of taste shocks, productivity shocks, and exogenous changes in government expenditures.
The slope parameter,
\[
\kappa = \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha} \frac{\phi^{-1} + \omega}{1 + \omega \theta} > 0,
\]
is a function of the structure of the economy, where \(\omega > 0\) denotes the elasticity of a firm’s real marginal cost with respect to its own output level. \(\theta > 1\) is the price elasticity of demand substitution among differentiated goods produced by firms in monopolistic competition. Each period, a share \(\alpha \in (0, 1)\) of randomly picked firms cannot adjust their prices, while the remaining \((1 - \alpha)\) firms get to choose prices optimally.

In addition, the exogenous shocks are assumed to follow AR(1) stochastic processes,
\[
\begin{align*}
\rho^n_t &= (1 - \rho_r) r_{ss} + \rho_r r^n_{t-1} + \sigma_{\varepsilon r} \varepsilon_{rt} \\
u_t &= \rho_u u_{t-1} + \sigma_{\varepsilon u} \varepsilon_{ut},
\end{align*}
\]
with first-order autocorrelation parameters \(\rho_j \in (-1, 1)\) for \(j = r, u\). The steady-state real interest rate \(r_{ss}\) is equal to \(1/\beta - 1\), such that \(r_{ss} \in (0, +\infty)\). And \(\sigma_{\varepsilon j} \varepsilon_{jt}\) are the innovations that buffet the economy, which are independent across time and cross-sectionally, and normally distributed with mean zero and standard deviations \(\sigma_{\varepsilon j} > 0\) for \(j = r, u\).

### 2.2 Monetary policy

In this basic model, I consider a range of monetary policy frameworks, with nominal policy rates constrained by the ZLB. After describing a typical benchmark policy, I introduce a nominal GDP level target in optimal discretionary policies and in simple policy rules.

#### 2.2.1 Ramsey plan

As a benchmark in the policy evaluation, I use the optimal Ramsey plan, i.e., the optimal commitment policy determined at time zero. The policymaker’s objective function in this case is the social welfare function:
where $\lambda$ denotes the weight assigned to stabilizing real GDP relative to inflation. This objective function, as explained by Woodford (2010), can be derived as a second-order approximation of the lifetime utility function of the representative household. The utility function is validly approximated around zero inflation. The approximation of the utility function allows to determine $\lambda$ in terms of the structure of the model economy. Thus, $\lambda$ is equal to $\kappa/\theta$ in this model.

### 2.2.2 Optimal discretionary policies

Under optimal discretion, the policymaker does not commit to the Ramsey plan and instead re-optimizes in each period, as described in Woodford (2010). I focus on two monetary policy frameworks in such a setting, namely inflation targeting and nominal GDP level targeting.

First, with inflation targeting under discretion, the policymaker’s objective function takes the form:

$$
\min_{\{\pi_t, x_t, \lambda_t \geq 0\}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[ \pi_t^2 + \lambda_t x_t^2 \right],
$$

where $\lambda$ denotes the weight assigned to stabilizing real GDP relative to inflation. If the weight is zero, the central bank is labeled a strict inflation targeter. But if the weight is positive, the central bank is then a flexible inflation targeter. $\pi^*$ denotes the inflation target pursued by the central bank, as part of its legislative mandate to achieve price stability. And $x^*$ is the resulting real GDP gap target.$^4$

Raising $\pi^*$ limits the incidence of hitting the ZLB. If $\pi^*$ is below a critical value, a bad shock can push the economy into a disastrous deflationary spiral from which there is no escape. The representative household then suffers an infinite welfare loss, because inflation variability and real GDP variability are unboundedly large. However, as I argued in Billi (2011), rasing

$^4$Phillips curve (2) implies $x^* = (1 - \beta) \kappa^{-1} \pi^*$. 
\(\pi^*\) above the critical value ensures that the economy reverts to a stable equilibrium rather than falling into a deflationary trap. But in that paper I did not consider the anchoring effect of a nominal GDP level target.

As a second framework for monetary policy under discretion, I consider nominal GDP level targeting. The policymaker’s objective function becomes:

\[
\min_{(y_t, i_t \geq 0)} E_t \sum_{j=0}^{\infty} \beta^j (y_{t+j} - \bar{y}_{t+j})^2, \tag{5}
\]

where \(y_t\) denotes the nominal GDP gap, which is equal to \(p_t + x_t\). \(\bar{y}_t\) denotes the corresponding target, which is assumed to increase at a deterministic rate \(\bar{p}_t + x^*\). And \(\bar{p}_t\) is the corresponding price level target which increases at rate \(\pi^*\), such that \(\bar{p}_t = \bar{p}_{t-1} + \pi^*\). Raising \(\pi^*\) serves as prevention against a deflationary spiral. But the scope for prevention is limited, because of the anchoring effect of the nominal GDP level target.

### 2.2.3 Simple policy rules

In addition to the optimal policies, I focus on two simple policy rules with forward guidance. In this case, forward guidance involves adjustments to the setting of the central bank’s policy interest rate. I consider in particular a Taylor-type rule and a nominal GDP level rule.

First, along the lines of Taylor and Williams (2010), I consider a version of the Taylor rule subject to the ZLB constraint:

\[
i_t = \max \left[ 0, \phi_p (\pi_t - \pi^*) + \phi_x (x_t - x^*) + (1 - \phi_i) i^* + \phi_i i_{t-1}^{\text{ss}} \right], \tag{6}
\]

where \(\phi_p\) and \(\phi_x\) are positive response coefficients on the inflation gap and real GDP gap, respectively. \(i^*\) denotes the equilibrium nominal policy rate, which is equal to \(r_{ss} + \pi^*\). In addition, the rule incorporates forward policy guidance on the nominal interest rate, or smoothing in the behavior of the interest rate, through a positive value of the coefficient \(\phi_i\). But as argued, for example, by Taylor and Williams (2010) and Giannoni (2014), values of \(\phi_i\) above 1 would imply unusual behavior by the central bank. Thus, precisely as in those papers,
this simple rule allows smoothing to determine the long-term response of policy. As a result, the central bank behaves to the extent possible as under the optimal commitment policy, as stressed, for example, by Woodford (2010) and Giannoni (2014).

Moreover, as in Taylor and Williams (2010), \( i_{t-1}^u \) denotes an unconstrained policy rate, i.e., the preferred setting of the policy rate in the previous period that would occur absent the ZLB. Thus, the policy interest rate is kept below its equilibrium value following an episode when the ZLB is a binding constraint on policy. This approach implies that the central bank compensates for the lost monetary stimulus due to the existence of the ZLB, even though the central bank does not use forward guidance on nominal GDP.

As a second simple rule for monetary policy, I consider a \textit{nominal GDP level rule} with ZLB constraint:

\[
i_t = \max \left[ 0, \phi_y (y_t - \bar{y}_t) + (1 - \phi_i) i^* + \phi_i i_{t-1}^u \right],
\]

where \( \phi_y \) is the coefficient on the nominal GDP gap and \( \phi_i \) is the smoothing coefficient. If the value of \( \phi_i \) is zero, the central bank uses forward guidance on the nominal GDP level only, to compensate for the lost monetary stimulus due to the existence of the ZLB. But if \( \phi_i \) is positive, the central bank uses forward guidance both on the nominal GDP level and nominal policy rate. Of course, the scope for smoothing the nominal policy rate is limited, because the nominal GDP rule provides forward policy guidance even in the absence of smoothing. With this range of monetary policy frameworks, I can study the effects of forward guidance on monetary policy performance.

### 2.3 Measuring welfare

Next, I need a criterion to evaluate monetary policy performance in this model. As argued by Woodford (2009), objective function (3) in this setting is the right criterion because it includes

\footnote{In the model, \( i_{t}^u \) is an observable variable because it is a function of observables only. But in actuality, the equilibrium real interest rate and the GDP gap are unobservables, which implies a challenge for the implementation of policy. Of course, a simple way to address this challenge is to base policy on prices only, ignoring the equilibrium rate and GDP gap altogether in setting policy.}
the costs of steady-state inflation. To clarify, the welfare loss associated with inflation in period $t$ includes two parts:

$$E_0 [\pi_t^2] = \pi^*^2 + E_0 [(\pi_t - \pi^*)^2].$$

On the right side, the first term is the welfare loss due to steady-state inflation, and the second term is the loss due to inflation variability. Both parts are relevant for a correct policy evaluation in this model in the presence of the ZLB. Neglecting, in particular, the first term would imply that welfare is strictly increasing in $\pi^*$, because raising $\pi^*$ limits the incidence of hitting the ZLB. This is the case, for example, in Coibion, Gorodnichenko, and Wieland (2012) when they linearize the model around zero inflation.\footnote{In fact, in their figure 2A, the dash line is strictly increasing.}

### 2.4 Equilibrium

In equilibrium, the policymaker chooses a policy based on a response function $y(s_t)$ and a state vector $s_t$. The corresponding expectations function takes the form:

$$E_t y_{t+1} (s_t) = \int y(s_{t+1}) f (\varepsilon_{t+1}) d (\varepsilon_{t+1}),$$

where $f (\cdot)$ is a probability density function of the future innovations that buffet the economy. Because there is uncertainty about the future state of the economy, the ZLB is an occasionally-binding constraint among the endogenous variables in the model.

In such a setting, I provide the following equilibrium definition:

**Definition 1 (SREE)** A stochastic, rational-expectations equilibrium is given by a response function and corresponding expectations function, $y(s_t)$ and $E_t y_{t+1} (s_t)$, respectively, which satisfy the equilibrium conditions, derived in Appendix A.1.

Ignoring the existence of uncertainty about the evolution of the economy, the model could be solved with a standard numerical method. This is the case, for example, in Coibion, Gorodnichenko, and Wieland (2012). By contrast, as in Billi (2011), I use a numerical procedure
that accounts for the ZLB and uncertainty about the future state of the economy. When the ZLB threatens, the mere possibility of hitting the ZLB causes expectations of a future decline in GDP below potential and inflation below its target, as showed in Adam and Billi (2006, 2007) and Nakov (2008). But those papers did not consider the anchoring effect of forward guidance.

2.5 Calibration

Before turning to the policy evaluation, I calibrate the model to U.S. data. The baseline calibration is shown in table 1. Specifically, I use the parameter values in Giannoni (2014) but with a more persistent real-rate shock as in previous research on the ZLB. Overall, the baseline is very similar to the one I used in Billi (2011) if price indexation is set to zero. Of course, the likelihood of a ZLB episode depends on the variance in the real-rate shock process. I, thus, depart from the baseline and consider a range of values for the shock variance, in section 3.3, to check the robustness of the findings.

3 Policy evaluation

Employing the small New Keynesian model with a plausible calibration to U.S. data, I evaluate the optimal policies and simple policy rules. In the evaluation, I take into account that the nominal policy rate occasionally hits the ZLB. After illustrating the anchoring effect of a nominal GDP level target, I show that forward guidance on the policy interest rate anchors inflation even more firmly. Finally, I show this result to be robust.

7As stressed in Billi (2011), price indexation fuels a deflationary spiral once the ZLB has been reached, and weakness in the economy puts downward pressure on inflation. In such a setting, the central bank has an even stronger incentive to give guidance directly on interest rates, rather than indirectly on a level of GDP it seeks to achieve, to counter downward pressure on inflation.
3.1 Optimal policies

The starting point for the evaluation is the performance of the optimal policies. I show that under optimal discretion the economy can fall into a disastrous deflationary trap, in which there is no stable equilibrium and the representative household suffers an infinite welfare loss. There are ways, however, to prevent a deflationary trap.

One way is to raise the inflation target above a critical value. To illustrate, figure 1 shows the welfare loss, measured as the permanent consumption loss relative to the Ramsey plan, as a function of the inflation target.\(^8\) In the top panel, the critical value of the inflation target is between 1.6 percent annual (strict inflation targeting) and 1.8 percent annual (flexible inflation targeting). The flexible inflation targeter is assumed to assign an optimal weight to stabilizing real GDP relative to inflation.\(^9\) Below the critical value, the inflation target is not high enough to ensure that a stable equilibrium exists toward which the economy tends to revert. Because inflation and real GDP variability are unboundedly large, the representative household suffers an infinite welfare loss.

\[\text{[Figure 1 about here]}\]

At the critical value of the inflation target, however, the welfare loss relative to the Ramsey plan is minimized or, conversely, welfare is maximized for the representative household in the economy. The intuition for why the critical value is slightly lower with strict inflation targeting is straightforward. If inflation targeting is strict, inflation is less variable and therefore more tightly anchored to the target. As a result, the economy is less vulnerable to a deflationary spiral from which there is no escape. On the other hand, by focusing entirely on stabilizing inflation, real GDP is more variable than would otherwise be the case. With strict inflation targeting, the welfare loss at the critical value amounts to 0.85 percent of permanent consumption, but declines to 0.79 percent of permanent consumption in the case of flexible inflation.

\(^8\)I first obtain the value of objective function (3) by averaging across 10,000 stochastic simulations each 1,000 periods long after a burn-in period. I then convert this value into a permanent consumption loss, as explained in Appendix A.2.

\(^9\)I find that in the case of optimal discretion, welfare is maximized with a weight \(\lambda^d\) of 0.001. I, thus, use this optimal weight in the analysis. Of course, this weight is smaller than the corresponding \(\lambda\) of 0.003 in the Ramsey plan, because lack of commitment causes a stabilization bias. To reduce the stabilization bias, the central bank has to focus more on stabilizing inflation compared to the Ramsey plan.
targeting. Thus, on balance, flexible inflation targeting is preferable in terms of welfare to strict inflation targeting.

Though raising the inflation target helps to avoid the deflationary trap, another way is to adopt nominal GDP level targeting. In this case, as the bottom panel of figure 1 shows, even if the inflation target is zero, inflation remains firmly anchored and therefore the economy does not fall into the deflationary trap. However, an inflation target slightly above zero minimizes the welfare loss relative to the Ramsey plan. As the bottom panel shows, with nominal GDP level targeting, the optimal inflation target that minimizes the welfare loss is 0.2 percent annual. With nominal GDP level targeting, the welfare loss at the optimal inflation target amounts to about 0.08 percent of permanent consumption, which represents a substantial decline in the welfare loss compared to the inflation targeting cases depicted in the top panel. In fact, the scale of the welfare loss is one order of magnitude smaller in the bottom panel than in the top panel. Thus, nominal GDP level targeting is clearly preferable in terms of welfare to inflation targeting. Furthermore, in contrast to inflation targeting, nominal GDP level targeting provides a firm nominal anchor to the economy.

During a ZLB episode, in particular, nominal GDP level targeting allows inflation to temporarily rise above its target. This bout of inflation implies a speedier economic recovery than would otherwise be the case. The reason is that the surge in prices encourages firms to expand production. The ability to jump start the economic recovery and push inflation above target are salient features of the Ramsey plan. However, the inflation targeter under discretion lacks the resolve to push inflation above target. To illustrate, figure 2 shows the expected evolution of the model economy after a –2 standard deviation real-rate shock.\textsuperscript{10} Shown are the expected paths under the optimal discretionary policies, with optimal inflation targets that minimize the welfare loss relative to the Ramsey plan. Also shown is the expected path in the case of the Ramsey plan. With the Ramsey plan, the nominal policy rate is gradually

\textsuperscript{10}I obtain the expected paths by averaging across 10,000 stochastic simulations. In comparing the policy rate paths in the top panel of figure 2, one has to keep in mind that the equilibrium levels are not the same. In the case of inflation targeting, the equilibrium level of the nominal policy rate is clearly higher. The reason is that the optimal inflation target is noticeably higher with inflation targeting than with nominal GDP level targeting, as shown in figure 1.
raised back to its equilibrium level during the economic recovery (top panel). This prolonged monetary stimulus causes real GDP to rise above potential (middle panel) and inflation to rise above its target (bottom panel). Also in the case of nominal GDP level targeting, prolonged monetary stimulus pushes inflation above target, albeit to a lesser extent than in the Ramsey plan. But with flexible inflation targeting, inflation does not rise above its target during the economic recovery.

Nominal GDP level targeting is, therefore, more effective at stabilizing the economy than inflation targeting. Table 2 summarizes the performance of the discretionary policies, with optimal inflation targets that minimize the welfare loss relative to the Ramsey plan. The table reports the expected frequency and duration of ZLB episodes. It also reports the welfare loss relative to the Ramsey plan. With the Ramsey plan, the nominal policy rate is expected to hit the ZLB about 13 percent of the time. The incidence of hitting the ZLB remains 13 percent with nominal GDP level targeting but edges down to 10 percent with flexible inflation targeting.\textsuperscript{11} The lower incidence of hitting the ZLB is associated with higher inflation targets and, of course, higher costs of steady-state inflation. With flexible inflation targeting, the welfare loss due to steady-state inflation is 0.2 percent of permanent consumption. In addition, the welfare loss due to inflation variability is 0.49 percent of permanent consumption. And the welfare loss due to real GDP variability is 0.1 percent of permanent consumption. In sum, with flexible inflation targeting, the total welfare loss relative to the Ramsey plan is 0.79 percent of permanent consumption. By contrast, with nominal GDP level targeting, the total welfare loss amounts to only 0.08 percent of permanent consumption. Thus, in the case of optimal discretion, nominal GDP level targeting performs clearly better than inflation targeting.

\textsuperscript{11}At the same time, the expected duration of a ZLB episode under the two policy frameworks is roughly the same, or 2 consecutive quarters.
3.2 Simple policy rules

As the next step in the evaluation, I study the performance of the simple policy rules and the role of forward guidance in the rules. I show that forward guidance on the policy interest rate anchors inflation firmly.

To study the role of forward guidance, I first search numerically for the optimal rule coefficients and inflation targets that minimize the welfare loss relative to the Ramsey plan. Figure 3 shows the welfare loss, measured as the permanent consumption loss relative to the Ramsey plan, as a function of the rule coefficients. In each panel a single rule coefficient is changed, while the other rule coefficients and the inflation target are at their optimal values that minimize the welfare loss.\textsuperscript{12} Thus, each panel illustrates the marginal effect of a single rule coefficient on the welfare of the representative household in this model economy.

[Figure 3 about here]

As the top-left panel shows, the smoothing coefficient in the Taylor rule provides the greatest marginal effect on welfare. In fact, in the case of the Taylor rule, the welfare loss falls from 0.17 to about 0.02 percent of permanent consumption when the smoothing coefficient is raised from 0.7 to 1.\textsuperscript{13} But the smoothing coefficient has a smaller effect on welfare in the nominal GDP level rule, as the welfare loss only falls from about 0.03 to 0.01 of permanent consumption. The scope for smoothing is, of course, greater in the Taylor rule than in the nominal GDP rule, because the nominal GDP rule provides forward policy guidance even in the absence of smoothing. As the top-right panel shows, the coefficient on the inflation gap in the Taylor rule bares almost no visible effect on welfare, even though the coefficient ranges widely between 0.1 to 1. This occurs because there is a tension between stabilizing inflation

\textsuperscript{12}This figure highlights the computational challenge in the analysis. Obtaining the results in the figure requires searching for the optimal inflation target for each combination of the rule coefficients in each of the rules. In the case of the Taylor rule, for example, there are three rule coefficients and therefore the search is over a four-dimensional parameter space. For each parameter combination, the model has to be solved and then stochastic simulations allow to obtain the welfare loss. But a single step in the search process can take hours on a workstation. To address this computational challenge, I thus resort to high performance computing.

\textsuperscript{13}In other terms, the figure shows that raising the smoothing coefficient in the Taylor rule from 0.7 to 1 provides a welfare gain of about 0.15 percent of permanent consumption, when the other rule coefficients and the inflation target are at their optimal values.
and real economic activity. And the bottom panels show that, the coefficients on nominal and real GDP gaps in the rules have a moderate effect on welfare, as the welfare loss falls by roughly 0.06 percent of permanent consumption. This last result highlights a role for policy to react to a measure of economic activity, because the GDP gap is an indicator of future inflationary pressures in the economy.

In this model, the optimal rule coefficients are in practice equal to 1. As the various panels in figure 3 show, raising each of the rule coefficients towards 1 leads to a decline in the welfare loss or, in other terms, a welfare improvement for the representative household in the economy. But the higher the rule coefficients, the smaller the welfare improvement. This occurs because with a strong policy response, the welfare loss as a function of the rule coefficients becomes practically flat. As a result, raising the rule coefficients above 1 (not shown in the figure) would not lead to a noticeable, further improvement in welfare for the representative household in the economy.

With the optimal rule coefficients, I can now proceed to illustrate the anchoring effect of forward guidance. To do so, figure 4 shows the welfare loss, measured as the permanent consumption loss relative to the Ramsey plan, as a function of the inflation target. Shown are three cases, in which the central bank respectively uses forward guidance on nominal GDP only, policy rates only, or both. In each case, the rule coefficients are at their optimal values that minimize the welfare loss relative to the Ramsey plan. As the figure shows, with forward guidance on nominal GDP only, the optimal inflation target that minimizes the welfare loss is 0.2 percent annual and the corresponding welfare loss is about 0.12 percent of permanent consumption. But with forward guidance on policy rates only, inflation is more firmly anchored in the economy. In fact, the optimal inflation target edges down to zero and the corresponding welfare loss falls below 0.02 percent of permanent consumption. Finally, with forward guidance both on nominal GDP and policy rates, there is a slight welfare improvement.\footnote{This result is not surprising. In fact, the Taylor rule with a smoothing coefficient of 1 resembles closely the nominal GDP level rule with a smoothing coefficient of zero, were it not for the response to the output gap, as argued, for example, by Giannoni (2014).} Still, the scope for improvement is clearly limited, because forward guidance on policy rates already provides
a firm nominal anchor to the economy.

Forward guidance in the rules makes inflation temporarily rise above target during a ZLB episode. This surge in inflation promotes a more rapid economic recovery than can be achieved otherwise. To illustrate, figure 5 shows the expected evolution of the model economy after a –2 standard deviation real-rate shock. Shown are the expected paths under the simple policy rules, with optimal coefficients and inflation targets that minimize the welfare loss relative to the Ramsey plan. With forward guidance on policy rates, the nominal policy rate is gradually raised back to its equilibrium level during the economic recovery. This prolonged monetary stimulus causes real GDP to rise above potential and inflation to rise above its target. But with forward guidance on nominal GDP only, the shape of the economic recovery is quite different. In particular, because of the limited amount of monetary stimulus, the recession is deeper and the economic recovery is slower.

Forward guidance on policy rates is, therefore, more effective in stabilizing the economy than forward guidance on nominal GDP. Table 3 summarizes the performance of the simple policy rules, with optimal coefficients and inflation targets that minimize the welfare loss relative to the Ramsey plan. As the table shows, compared to forward guidance on nominal GDP, forward guidance on policy rates is associated with a lower inflation target and therefore a higher incidence of hitting the ZLB. It is also associated with a lower welfare loss. As the table shows, with forward guidance on nominal GDP, the total welfare loss rises to 0.12 percent of permanent consumption, which is 0.1 percent higher than with forward guidance on policy rates. Thus, forward guidance on policy rates performs clearly better than forward guidance on nominal GDP.
3.3 Robustness checks

As the final step in the policy evaluation, I show the robustness of the anchoring effect of forward guidance to the shock calibration and monetary policy framework.

The anchoring effect of forward guidance depends on the variance in the real-rate shock process, because a higher shock variance increases the likelihood of a ZLB episode. Figure 6 shows the welfare loss, measured as the permanent consumption loss relative to the Ramsey plan, as a function of the standard deviation of the real-rate shock process. The standard deviation is raised from its baseline value of 0.75 to 0.1 percent, which implies a substantial one-third increase in the standard deviation. As a consequence, as the top-left panel shows, the welfare loss increases sharply from about 0.12 to 0.45 percent of permanent consumption in the case of forward guidance on nominal GDP, while there is no noticeable change in welfare in the case of forward guidance on policy rates. In particular, as the other panels show, the welfare deterioration with forward guidance on nominal GDP is due to an increase in both inflation variability and real GDP variability. In sum, the anchoring effect of forward guidance on nominal GDP is prone to a substantial deterioration if there is greater uncertainty about the evolution of the economy. By contrast, the anchoring effect of forward guidance on policy rates is very robust to uncertainty.

[Figure 6 about here]

In the case of forward guidance on nominal GDP, the central bank reacts with the same intensity to prices and real economic activity. But a more flexible approach is to react with different intensity. I, thus, consider a price level rule subject to the ZLB constraint:

\[
i_t = \max \left[ 0, \phi_p (p_t - \bar{p}_t) + \phi_x (x_t - x^*) + (1 - \phi_i) i^* + \phi_i i_{t-1} \right],
\]

where \( \phi_p \) and \( \phi_x \) are coefficients on the price level gap and real GDP gap, respectively. \( \phi_i \) is the smoothing coefficient. If the value of \( \phi_x \) is zero, this is labeled a strict price level rule. But if \( \phi_x \) is positive, this is a flexible price level rule. In particular, when \( \phi_x \) is assumed to be equal to \( \phi_p \), rule (8) coincides with rule (7), as assumed until this point in the analysis.
To study the price level rule, figure 7 shows the welfare loss, measured as the permanent consumption loss relative to the Ramsey plan, as a function of the rule coefficients. In each panel a single rule coefficient is changed, while the other rule coefficients and the inflation target are at their optimal values that minimize the welfare loss. With the flexible price level rule, as the top panels show, the optimal values of both $\phi_p$ and $\phi_x$ are in practice equal to 1, just as found before in the analysis. As before, the welfare loss with optimal coefficients is about 0.01 percent of permanent consumption. But with the strict price level rule, as the bottom panels show, the optimal value of $\phi_p$ rises sharply from 1 to 10. The reason is that the central bank has to focus more intensely on current prices, given that it no longer reacts to an indicator of future inflationary pressures in the economy. For the same reason, the optimal value of $\phi_i$ declines to 0.2. As a consequence of focusing on prices only, the welfare loss increases to about 0.09 percent of permanent consumption. As a result, the analysis is robust to the consideration of the price level rule.

[Figure 7 about here]

4 Conclusion

In this article, I shed light on recent proposals directed at major central banks to abandon inflation targeting and instead adopt nominal GDP level targeting. According to the New Keynesian literature, nominal GDP level targeting is clearly a superior framework for monetary policy compared to inflation targeting. In this article, I test the two targets under central bank discretion and simple policy rules with forward guidance, in a small New Keynesian model subject to the ZLB constraint. The stylized model allows for a clear intuition about the key tradeoffs faced by policymakers under the two targets.

I show that, for a plausible calibration of the model to U.S. data, an inflation target outperforms a nominal GDP target, because it allows the central bank to give guidance directly on interest rates, rather than indirectly on a level of GDP it seeks to achieve. With forward

\footnote{With $\phi_p$ and $\phi_x$ equal to 1, the effect of $\phi_i$ on welfare can be seen already in the top-left panel of figure 3.}
guidance on interest rates, inflation is more tightly anchored than would otherwise be the case. Furthermore, the greater the uncertainty about the evolution of the economy, the worse the performance of a nominal GDP target. Thus, the analysis suggests that, contrary to some recent proposals, inflation targeting should clearly not be ditched.

Nevertheless, one could argue that central banks should adopt nominal GDP level targeting, and thus use forward guidance both on nominal GDP and policy rates. Of course, this line of reasoning implies a fundamental change in the current monetary policy framework of major central banks. But the analysis does not take into account any potential costs associated with a change in the policy framework, such as the uncertainty faced by households and firms about the occurrence of a regime change. Accounting for the costs of regime change, the central bank would have an even stronger incentive to give guidance directly on interest rates, to anchor inflation firmly in the economy.

A Appendix

A.1 Equilibrium conditions

I first derive the Kuhn-Tucker conditions that close the model in the case of optimal policies. With simple policy rules instead, the rule itself closes the model. Next, I summarize the equilibrium conditions in a table.

**Ramsey plan.** The Lagrangian of problem (1)-(3) is

\[
\max_{\{\pi_t, x_t, u_t \geq 0\}_{t=0}^{\infty}} \min_{(m_{1t}, m_{2t})_{t=0}^{\infty}} \mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -\pi_t^2 - \lambda x_t^2 \right. \\
+ m_{1t} \left[ \pi_t - \kappa x_t - u_t \right] - m_{1t-1} \pi_t \\
+ m_{2t} \left[ -x_t - \varphi (i_t - r^n_t) \right] + m_{2t-1} \beta^{-1} (x_t + \varphi \pi_t) \left. \right\}.
\]

The Kuhn-Tucker conditions of this problem are
\[ \frac{\partial L}{\partial \pi_t} = -2\pi_t + m_{1t} - m_{1t-1} + \beta^{-1}\varphi m_{2t-1} = 0 \]  
(9)

\[ \frac{\partial L}{\partial x_t} = -2\lambda x_t - \kappa m_{1t} - m_{2t} + \beta^{-1}m_{2t-1} = 0 \]  
(10)

\[ \frac{\partial L}{\partial i_t} \cdot i_t = -\varphi m_{2t} \cdot i_t = 0, \quad m_{2t} \geq 0, \quad i_t \geq 0. \]  
(11)

**Inflation targeting under discretion.** The Lagrangian of problem (1), (2) and (4) is

\[
\max_{(\pi_t,x_t,i_t \geq 0)(m_{1t},m_{2t})} \min L \equiv E_t \sum_{j=0}^{\infty} \beta^j \left\{ - (\pi_{t+j} - \pi^*)^2 - \lambda^d (x_{t+j} - x^*)^2 
+ m_{1t+j} [\pi_{t+j} - \beta E_{t+j}\pi_{t+j+1} - \kappa x_{t+j} - u_{t+j}]
+ m_{2t+j} [-x_{t+j} + E_{t+j}x_{t+j+1} - \varphi (i_{t+j} - E_{t+j} \pi_{t+j+1} - r_n^{u+j})] \right\} \\
\text{and } \{y(s_{t+j})\} \text{ given for } j \geq 1.
\]

The Kuhn-Tucker conditions are

\[ \frac{\partial L}{\partial \pi_t} = -2 (\pi_t - \pi^*) + m_{1t} = 0 \]  
(12)

\[ \frac{\partial L}{\partial x_t} = -2\lambda^d (x_t - x^*) - \kappa m_{1t} - m_{2t} = 0 \]  
(13)

\[ \frac{\partial L}{\partial i_t} \cdot i_t = -\varphi m_{2t} \cdot i_t = 0, \quad m_{2t} \geq 0, \quad i_t \geq 0. \]  
(14)

**Nominal GDP level targeting under discretion.** To solve for a stationary equilibrium, I use the price level in deviation from its trend

\[ \hat{p}_t \equiv p_t - \hat{p}_t = \hat{p}_{t-1} + \pi_t - \pi^*. \]  
(15)

Using this identity, the Lagrangian of problem (1), (2), (5) and (15) can be written as
The Kuhn-Tucker conditions are

\[
\begin{align*}
\partial L / \partial \pi_t &= -2(\hat{p}_t + x_t - x^*) + m_{1t} + m_{3t} = 0 \quad (16) \\
\partial L / \partial x_t &= -2(\hat{p}_t + x_t - x^*) - \kappa m_{1t} - m_{2t} = 0 \quad (17) \\
\partial L / \partial i_t \cdot i_t &= -\varphi m_{2t} \cdot i_t = 0, \quad m_{2t} \geq 0, \quad i_t \geq 0 \quad (18) \\
\partial L / \partial \hat{p}_t &= -2(\hat{p}_t + x_t - x^*) - (\beta m_{1t} - \varphi m_{2t}) \cdot \partial E_t \pi_{t+1} / \partial \hat{p}_t \\
&\quad + m_{2t} \cdot \partial E_t x_{t+1} / \partial \hat{p}_t - m_{3t} = 0. \quad (19)
\end{align*}
\]

A summary of the equilibrium conditions is provided in the following table:

<table>
<thead>
<tr>
<th>Policy framework</th>
<th>Equilibrium conditions</th>
<th>State vector ( s_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ramsey plan</td>
<td>(1), (2) and (9)-(11)</td>
<td>( (u_t, r^n_t, m_{1t-1}, m_{2t-1}) )</td>
</tr>
<tr>
<td>Inflation targeting (discretion)</td>
<td>(1), (2) and (12)-(14)</td>
<td>( (u_t, r^n_t) )</td>
</tr>
<tr>
<td>Nominal GDP targeting (discretion)</td>
<td>(1), (2) and (15)-(19)</td>
<td>( (u_t, r^n_t, \hat{p}_{t-1}) )</td>
</tr>
<tr>
<td>Taylor rule</td>
<td>(1), (2) and (6)</td>
<td>( (u_t, r^n_t, i^n_{t-1}) )</td>
</tr>
<tr>
<td>Nominal GDP level rule</td>
<td>(1), (2), (7) and (15)</td>
<td>( (u_t, r^n_t, i^n_{t-1}, \hat{p}_{t-1}) )</td>
</tr>
<tr>
<td>Price level rule</td>
<td>(1), (2), (8) and (15)</td>
<td>( (u_t, r^n_t, i^n_{t-1}, \hat{p}_{t-1}) )</td>
</tr>
</tbody>
</table>
A.2 Permanent consumption loss

I obtain the permanent consumption loss as in Billi (2011). The expected lifetime utility of the representative household is validly approximated by

\[ E_0 \sum_{t=0}^{\infty} \beta^t U_t = \frac{U_c \bar{C}}{2} \frac{\alpha \theta (1 + \omega \theta)}{(1 - \alpha) (1 - \alpha \beta)} L, \]  

(20)

where \( \bar{C} \) is steady-state consumption; \( U_c > 0 \) is steady-state marginal utility of consumption; and \( L \geq 0 \) is the value of objective function (3).

At the same time, a steady-state consumption loss of \( \mu \geq 0 \) causes a utility loss

\[ E_0 \sum_{t=0}^{\infty} \beta^t U_c \mu = \frac{1}{1 - \beta} U_c \bar{C} \mu. \]  

(21)

Equating the right sides of (20) and (21) gives

\[ \mu = \frac{1 - \beta}{2} \frac{\alpha \theta (1 + \omega \theta)}{(1 - \alpha) (1 - \alpha \beta)} L. \]

References


Table 1: Baseline calibration of the model

<table>
<thead>
<tr>
<th>Definition</th>
<th>Parameter</th>
<th>Numerical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Interest elasticity of aggregate demand</td>
<td>$\varphi$</td>
<td>6.25</td>
</tr>
<tr>
<td>Share of firms keeping prices fixed</td>
<td>$\alpha$</td>
<td>0.66</td>
</tr>
<tr>
<td>Price elasticity of demand</td>
<td>$\theta$</td>
<td>7.66</td>
</tr>
<tr>
<td>Elasticity of a firms’ marginal cost</td>
<td>$\omega$</td>
<td>0.47</td>
</tr>
<tr>
<td>Slope of aggregate supply curve</td>
<td>$\kappa$</td>
<td>0.024</td>
</tr>
<tr>
<td>Weight on real GDP gap (Ramsey plan)</td>
<td>$\lambda$</td>
<td>0.003</td>
</tr>
<tr>
<td>Steady-state real interest rate</td>
<td>$r_{ss}$</td>
<td>1.00 percent</td>
</tr>
<tr>
<td>Standard deviation of real-rate shock</td>
<td>$\sigma_r$</td>
<td>0.75 percent</td>
</tr>
<tr>
<td>Standard deviation of mark-up shock</td>
<td>$\sigma_u$</td>
<td>0.10 percent</td>
</tr>
<tr>
<td>AR(1) parameter of real-rate shock</td>
<td>$\rho_r$</td>
<td>0.80</td>
</tr>
<tr>
<td>AR(1) parameter of mark-up shock</td>
<td>$\rho_u$</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Note: Because in the model a period is one quarter, shown are parameter values corresponding to inflation and interest rates measured at a quarterly rate.
Table 2: Performance of optimal policies

<table>
<thead>
<tr>
<th>Policy frameworka</th>
<th>Inflation targetb</th>
<th>ZLB episodes</th>
<th>Welfare loss relative to Ramseye</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\pi^*) freq.c</td>
<td>durationd</td>
<td>steady state</td>
</tr>
<tr>
<td>Discretion</td>
<td></td>
<td></td>
<td>(\pi)</td>
</tr>
<tr>
<td>Flexible inflation targeting</td>
<td>1.8 10 2</td>
<td>0.20 0.00</td>
<td>0.49 0.10</td>
</tr>
<tr>
<td>Nominal GDP targeting</td>
<td>0.2 13 2</td>
<td>0.00 0.00</td>
<td>0.04 0.04</td>
</tr>
<tr>
<td>Ramsey plan</td>
<td>0 13 3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. If flexible inflation targeting, optimal weight \(\lambda^d = 0.001\)

b. Percent annual
c. Expected percent of time at the ZLB
d. Expected number of consecutive quarters at the ZLB
e. Permanent consumption loss (percentage points)
Table 3: Performance of simple policy rules with forward guidance

<table>
<thead>
<tr>
<th>Forward guidance</th>
<th>Inflation target(^d)</th>
<th>ZLB episodes</th>
<th>Welfare loss relative to Ramsey(^g)</th>
<th>steady state</th>
<th>variability</th>
<th>Tot.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\pi^*)</td>
<td>freq.(^e)</td>
<td>duration(^f)</td>
<td>(\pi)</td>
<td>(x)</td>
<td>(\pi)</td>
</tr>
<tr>
<td>on nominal GDP only(^a)</td>
<td>0.2</td>
<td>9</td>
<td>2</td>
<td>0.00</td>
<td>0.00</td>
<td>0.07</td>
</tr>
<tr>
<td>on policy rate only(^b)</td>
<td>0.0</td>
<td>10</td>
<td>3</td>
<td>0.00</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>on both(^c)</td>
<td>0.0</td>
<td>11</td>
<td>4</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
</tr>
</tbody>
</table>

\(^a\) Rule (7) with \(\phi_\pi = 0\) and \(\phi_y = 1\)

\(^b\) Rule (6) with \(\phi_\pi = 1\), \(\phi_\pi = 1\) and \(\phi_x = 1\)

\(^c\) Rule (7) with \(\phi_\pi = 1\) and \(\phi_y = 1\)

\(^d\) Percent annual

\(^e\) Expected percent of time at the ZLB

\(^f\) Expected number of consecutive quarters at the ZLB

\(^g\) Permanent consumption loss (percentage points)
Figure 1: Optimal discretion: deflationary trap versus firmly anchored inflation

Notes: The welfare loss is measured as the permanent consumption loss relative to the Ramsey plan. The flexible inflation targeter assigns optimal weight to stabilizing real GDP relative to inflation.
Figure 2: ZLB episode with optimal policies

Notes: Shown is the expected path after a $-2$ standard deviation real-rate shock, with optimal inflation targets. The flexible inflation targeter assigns optimal weight to the stabilization of real GDP relative to inflation.
Figure 3: Optimal, simple policy rules with forward guidance

Notes: The welfare loss is measured as the permanent consumption loss relative to the Ramsey plan. In each panel a single rule coefficient is changed, while the other rule coefficients and the inflation target are at their optimal values.
Figure 4: Anchoring effect of forward guidance in simple policy rules

Notes: The welfare loss is measured as the permanent consumption loss relative to the Ramsey plan. The rule coefficients are at their optimal values.
Figure 5: ZLB episode and simple policy rules with forward guidance

Notes: Shown is the expected path after a –2 standard deviation real-rate shock. The rule coefficients and the inflation target are at their optimal values.
Figure 6: Anchoring effect of forward guidance and shock variability

Notes: The welfare loss is measured as the permanent consumption loss relative to the Ramsey plan. The rule coefficients and the inflation target are at their optimal values.
Figure 7: Optimal, simple price level rule

Notes: The welfare loss is measured as the permanent consumption loss relative to the Ramsey plan. In each panel a single rule coefficient is changed, while the other rule coefficients and the inflation target are at their optimal values.