A Note on Nominal GDP Targeting and the Zero Lower Bound*

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Abstract

I compare nominal GDP level targeting to strict price level targeting in a small New Keynesian model, with the central bank operating under optimal discretion and facing a zero lower bound on nominal interest rates. I show that, if the economy is only buffeted by purely temporary shocks to inflation, nominal GDP level targeting may be preferable because it requires the burden of the shocks to be shared by prices and output. But in the presence of persistent supply and demand shocks, strict price level targeting may be superior because it induces greater policy inertia and improves the tradeoffs faced by the central bank. During lower bound episodes, somewhat paradoxically, nominal GDP level targeting leads to larger falls in nominal GDP.

Keywords: nominal level targets, optimal discretionary policy, zero lower bound

JEL: E31, E52, E58

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1 Introduction

With policy interest rates constrained by the zero lower bound (ZLB) and a weak global economy, alternatives to the monetary policy frameworks of major central banks are being proposed.1 Some argue in favor of a nominal GDP level target, which is conceptually appealing because the central bank then commits to make up for past shortfalls in economic activity.2 Shedding light on such a proposal, this article shows that a better alternative, which ensures greater policy stimulus during ZLB episodes, may be a strict price level target.

The article compares the two targeting frameworks in a small New Keynesian model, with the central bank operating under optimal discretion and facing a zero lower bound on nominal interest rates. Before turning to the evaluation of the frameworks, the model is calibrated to recent U.S. data, with the conduct of monetary policy described by a simple rule often used in policy analysis. The frameworks are then ranked in terms of performance, based on the model’s social welfare function. In the model, three types of shocks buffet the economy. On the supply side of the model, technology shocks push output gaps and prices in the same direction, while cost-push shocks cause instead an inflation-output tradeoff. On the demand side of the model, adverse demand shocks and the ZLB constraint cause a tradeoff between current and future output, because it is desirable in a ZLB episode to promise to induce an economic expansion after the ZLB episode. The stylized model offers a clear illustration of such tradeoffs in the evaluation of the frameworks.

The model produces three main results, one for each type of shock. First, if the economy is only subject to technology shocks, nominal GDP level targeting is clearly the less effective framework because it fails to insulate the economy, while strict price level targeting neutralizes the effects of the shocks entirely. Second, if the economy is only buffeted by purely temporary

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1 This article adopts the standard practice of referring to a zero lower bound for nominal interest rates, but the recent experience with negative nominal interest rates in Denmark, Sweden, and the eurozone suggests the effective lower bound is somewhat below zero. See Svensson (2010) for a discussion.

shocks to inflation, nominal GDP level targeting may be preferable because it requires the burden of the shocks to be shared by prices and output, while strict price level targeting causes costly fluctuations in output. But if shocks to inflation are persistent, nominal GDP level targeting results in costly price fluctuations, and the two frameworks may be similarly effective in terms of social welfare. Third, faced by persistent demand shocks during ZLB episodes, nominal GDP level targeting may be an inferior framework, because it induces less policy inertia and, ironically, leads to larger falls in nominal GDP. Further, such a ranking of the two targeting frameworks is shown to be robust to a wide range of alternate calibrations.

In the New Keynesian literature, the desirability of a nominal level target when the ZLB is a constraint was stressed by Eggertsson and Woodford (2003) and Svensson (2003), before the financial crisis and great recession. Related, Svensson (1999), Vestin (2006) and Giannoni (2014) argued in favor of price level targeting versus inflation targeting in the absence of the ZLB constraint. Jensen (2002) showed that nominal income growth targeting fails to insulate the economy from technology shocks. In the aftermath of the crisis, Billi (2011) and Coibion, Gorodnichenko, and Wieland (2012) studied the optimal rate of inflation in the presence of the ZLB. In this article, the analysis is conducted in a stylized model that does not include the effects of positive trend inflation, nor balance sheet policies and fiscal spending, which involve additional tradeoffs for monetary policy. Thus, further study is needed to extend the results to a broader class of models.

Section 2 describes the model and baseline calibration to recent data. Section 3 presents the policy evaluation and considers a range of alternate calibrations. And Section 4 concludes. The Appendix contains technical details on the model solution.

2 The model

I use a small New Keynesian model as described in Woodford (2010), but with a nominal level target in the central bank’s objective function. I also describe the conduct of monetary policy with a version of the Taylor rule, to be used in the model calibration. At the same time, I
take into account that the nominal policy rate occasionally hits the ZLB. After describing the
features of an equilibrium that accounts for the ZLB and uncertainty about the evolution of
the economy, I calibrate the model to U.S. data.

2.1 Private sector

The behavior of the private sector is summarized by two log-linearized, structural equations,
namely an Euler equation and a Phillips curve, respectively describing the demand and supply
side of the economy. The equations of this basic model are linearized around zero inflation.

The Euler equation, which describes the representative household’s expenditure decisions,
is given by

\[ y_t = E_t y_{t+1} - \varphi (i_t - r - E_t \pi_{t+1} - v_t), \]

where \( E_t \) denotes the expectations operator conditional on information available at time \( t \). \( y_t \)
is output measured as the log deviation from trend. \( \pi_t \) is the inflation rate, the log difference
of prices between the current and previous period, \( p_t - p_{t-1} \). And \( i_t \geq 0 \) is the short-term
nominal interest rate, which is the instrument of monetary policy and is constrained by a ZLB.
\( r > 0 \) is the steady-state interest rate, with zero inflation in steady state. Thus, \( i_t - r - E_t \pi_{t+1} \)
is the real interest rate in deviation from steady state. \( \varphi > 0 \) is the interest elasticity of real
aggregate demand, capturing intertemporal substitution in household spending.

The demand shock \( v_t \) represents other spending, for example, government spending. Be-
cause of the ZLB constraint, the effects of the demand shock on the economy are asymmetric.
A positive demand shock can be countered entirely by raising the nominal interest rate, while
a large negative shock that leads to hitting the ZLB causes output and prices to fall.\(^3\) As a
result, in the model the central bank faces a tradeoff between current and future economic
activity. The reason for this tradeoff is that, during a ZLB episode, the central bank can lower
the real interest rate and stimulate economic activity today by credibly promising to induce

\(^3\)The fall in prices stems from the supply side of the economy, equation (2).
a surge in economic activity and inflation after the ZLB episode.\footnote{The promise is credible if the central bank commits to make up for past shortfalls from the target, as is the case under optimal discretion with a nominal level target.}

The Phillips curve, which describes the optimal price-setting behavior of firms, under staggered price changes à la Calvo, is given by

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t, \tag{2}
\]

where $\beta \in (0, 1)$ is the discount factor of the representative household, determined as $1/ (1 + r)$. $x_t \equiv y_t - y^n_t$ is the output gap. $y^n_t$ is the natural rate of output, or potential output, the output deviation from trend that would prevail in the absence of any price rigidities, which represents a technology shock. A positive technology shock implies slack in economic activity and downward pressure on prices, while a negative shock implies a strong economy and puts upward pressure on prices. Because this type of shock pushes output gaps and prices in the same direction, it does not entail any tradeoffs for the central bank in the model, as long as the nominal interest rate does not reach the ZLB.

And $u_t$ is a cost-push shock, or a mark-up shock resulting from variation over time in the degree of monopolistic competition between firms. The mark-up shock causes an inflation-output tradeoff for monetary policy in the model. The reason for this tradeoff is that, if prices decrease because of a negative mark-up shock the central bank can stimulate economic activity to place upward pressure on prices, while if prices increase due to a positive shock the central bank can discourage economic activity and put downward pressure on prices.

The slope parameter in the Phillips curve,

\[
\kappa = \frac{(1 - \alpha) (1 - \alpha \beta)}{\alpha} \varphi^{-1} + \omega \frac{1}{1 + \omega \theta} > 0,
\]

is a function of the structure of the economy, where $\omega > 0$ denotes the elasticity of a firm’s real marginal cost with respect to its own output level. $\theta > 1$ is the price elasticity of demand substitution among differentiated goods produced by firms in monopolistic competition.\footnote{The seller’s desired markup is $\theta / (\theta - 1)$.} Each
period, a share $\alpha \in (0, 1)$ of randomly picked firms cannot adjust their prices, while the remaining $(1 - \alpha)$ firms get to choose prices optimally.\(^6\)

The three types of exogenous shocks are assumed to follow AR(1) stochastic processes,

\[
y_t^n = \rho_y y_{t-1} + \sigma_{y\epsilon} \epsilon_{yt} \\
 u_t = \rho_u u_{t-1} + \sigma_{u\epsilon} \epsilon_{ut} \\
 v_t = \rho_v v_{t-1} + \sigma_{v\epsilon} \epsilon_{vt}
\]

with first-order autocorrelation parameters $\rho_j \in (-1, 1)$ for $j = y, u, v$. And $\sigma_{j\epsilon} \epsilon_{jt}$ are the innovations that buffet the economy, which are independent across time and cross-sectionally, and normally distributed with mean zero and standard deviations $\sigma_{j\epsilon} > 0$ for $j = y, u, v$.

The policy frameworks to be considered are evaluated based on the social welfare function, given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \pi_t^2 + \lambda (x_t - x^*)^2 \right],
\]

where $\lambda$ is the weight assigned to stabilizing the output gap relative to inflation. $x^*$ is the target level of the output gap, which stems from monopolistic competition and distortion in the steady state. Output subsidies are assumed to offset the monopolistic distortion so that the steady state is efficient, $x^* = 0$.\(^7\) This social welfare function, as explained by Woodford (2010), can be derived as a second-order approximation of the lifetime utility function of the representative household. The utility function is approximated around zero inflation. The approximation of the utility function allows to determine $\lambda$ in terms of the structure of the model economy. Thus, $\lambda$ is equal to $\kappa/\theta$ in this model.

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\(^6\)The implied duration between price changes is $1/(1 - \alpha)$.

\(^7\)As a result, in the analysis there is no inflation bias but a stabilization bias due to discretionary policy.
2.2 Monetary policy

The conduct of monetary policy is first described by a simple rule, to be used in the model calibration. It is then described by optimal discretion with a nominal level target in the central bank’s objective function.

The simple policy rule employed is a version of the Taylor rule subject to the ZLB constraint along the lines of Taylor and Williams (2010):

\[ i_t = \max \left[ 0, \phi_i i_{t-1}^u + (1 - \phi_i) \left( r + \phi_p \pi_t + \phi_y (y_t - y^n_t) \right) \right], \tag{4} \]

where \( \phi_p \) and \( \phi_y \) are positive response coefficients on inflation and the output gap, respectively. The rule incorporates smoothing in the behavior of the interest rate, through a positive value of the coefficient \( \phi_i \). And \( i_{t-1}^u \) denotes an unconstrained policy rate, the preferred setting of the policy rate in the previous period that would occur absent the ZLB. Thus, the policy interest rate is kept below its equilibrium value following an episode when the ZLB is a binding constraint on policy. This approach implies that the central bank compensates to some extent for the lost monetary stimulus due to the existence of the ZLB, even though the central bank does not commit to make up for past shortfalls from a nominal level target.

Under optimal discretion, the policymaker has an objective function rather than a simple rule and re-optimizes its policy decision in each period, as described in Woodford (2010). In such a setting, two monetary policy frameworks are considered. First, with strict price level targeting, the objective function takes the form:

\[ \min_{i_t \geq 0} E_t \sum_{j=0}^{\infty} \beta^j p^2_{t+j}, \]

where \( p_t \) is the price level, which is equal to \( p_{t-1} + \pi_t \). In this framework, the policymaker seeks to stabilize prices without concern for output stability, and therefore transfers the entire burden of shocks onto output. The framework involves inertia in the behavior of policy, because the current policy decision depends on the past price level. To ensure price stability, the real interest rate is raised above its equilibrium value after shocks that put upward pressure
on prices, while it is pushed below its equilibrium value following deflationary shocks.

Second, with **nominal GDP level targeting**, the objective function becomes:

$$\min_{n_t \geq 0} E_t \sum_{j=0}^{\infty} \beta^j n_{t+j}^2,$$

where $n_t$ is the (detrended) level of nominal GDP, the log sum of prices and output, $p_t + y_t$. In this framework, the policymaker seeks to stabilize both prices and output, as opposed to focusing entirely on price stability, and the policymaker thus requires the burden of shocks to be shared by prices and output. As a consequence, however, the current policy decision involves relatively less dependence on the past price level, and the policymaker acts less in accordance with a precommitment to price stability. To ensure nominal GDP stability, the real interest rate is raised following shocks that put upward pressure on nominal GDP, while it is lowered after shocks that put downward pressure on nominal GDP.

### 2.3 Equilibrium

In equilibrium, the policymaker chooses a policy based on a response function $y(s_t)$ and a state vector $s_t$. The corresponding expectations function takes the form:

$$E_t y_{t+1} = \int y(s_{t+1}) f(\varepsilon_{t+1}) d(\varepsilon_{t+1}),$$

where $f(\cdot)$ is a probability density function of the future innovations that buffet the economy. Because there is uncertainty about the future state of the economy, the ZLB is an occasionally-binding constraint among the endogenous variables in the model.

In such a setting, I provide the following equilibrium definition:

**Definition 1 (SREE)** A stochastic, rational-expectations equilibrium is given by a response function and corresponding expectations function, $y(s_t)$ and $E_t y_{t+1}$, respectively, which satisfy the equilibrium conditions, derived in Appendix A.1.

Ignoring the existence of uncertainty about the evolution of the economy, the model could be solved with a standard numerical method, as for example in Coibion, Gorodnichenko, and
Wieland (2012). By contrast, as in Billi (2011), I use a numerical procedure that accounts for the ZLB and uncertainty about the future state of the economy.\(^8\) When the ZLB threatens, the mere possibility of hitting the ZLB causes expectations of a future decline in output and inflation, as shown by Adam and Billi (2006, 2007) and Nakov (2008).

### 2.4 Calibration

The small model economy is calibrated to U.S. data for recent decades. To do so, monetary policy in the model is described by the inertial Taylor rule (4) that features prominently in Federal Reserve discussions. The values of the rule coefficients are taken from English, Lopez-Salido and Tetlow (2013), with \(\phi_y = 1.5\), \(\phi_y = 1/4\) (quarterly) and \(\phi_i = 0.85\). The rule thus accounts for smoothing in the behavior of the policy interest rate.

The values of the structural parameters of the model are standard in the related literature, and are the same ones used for example by Billi (2011) and Giannoni (2014), among others. Regarding the calibration of the shocks, the first-order autocorrelation parameters \(\rho_{y,v}\) are set to 0.8, to generate persistent effects on the economy. At the same time, the standard deviations of the technology shock and demand shock \(\sigma_{y,v}\) are set to 0.8 percent (quarterly), to try to replicate the volatility of output and nominal interest rates, respectively. And the standard deviation of the mark-up shock \(\sigma_u\) is set to 0.05 percent (quarterly), to match the inflation volatility in the data. This baseline calibration is summarized in table 1.

[Table 1 about here]

Overall, with the simple policy rule and baseline calibration, the model does a fairly good job at replicating the relevant features of U.S. data for the sample period 1984Q1-2014Q4, as table 2 shows.\(^9\) The model strikes a balance between replicating output volatility and

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\(^8\) See Appendix A.2 for a description of the algorithm.

\(^9\) The inflation rate is measured as the continuously compounded rate of change in the seasonally adjusted, personal consumption expenditures chain-type price index less food and energy (source BEA). Output is measured as the log deviation from trend in seasonally adjusted, real gross domestic product (source BEA). And the nominal interest rate is measured as the average effective federal funds rate (source Fed Board). Because the funds rate is on average about 4 percent annual over the sample period, the discount factor \(\beta\) is set to 0.99 in the baseline.
nominal interest rate volatility in the data, and thus slightly overstates output volatility and understates nominal interest rate volatility. In fact, because in this basic model the nominal interest rate is the only available policy instrument, the model does not account for other policies used in actuality to stabilize output such as balance sheet policies and fiscal spending. Related, output and inflation are somewhat less persistent in the model results relative to the data, because this basic model for sake of simplicity does not allow for structural propagation mechanisms that give rise to output and inflation inertia.\textsuperscript{10}

[Table 2 about here]

Estimating the shocks with a similar model, Adam and Billi (2006) find that mark-up shocks do not display any significant autocorrelation, because of the estimation procedure that allows inflation to inherit the persistence of its prediction and of a shorter sample period that excludes the recent decade of low and stable inflation. But in this analysis, if mark-up shocks are assumed to have no persistence, the autocorrelation of inflation falls from its baseline value of 0.53 to 0.35 (not shown), which is then further below the autocorrelation of 0.78 in the data (table 2). However, the policy evaluation in the next section illustrates the importance of shock persistence for the model results, while the policy ranking with persistent shocks is shown to be robust to a wide range of alternate calibrations.

3 Policy evaluation

Employing the small New Keynesian model with a calibration to recent U.S. data, I compare the performance of nominal GDP level targeting and strict price level targeting under optimal discretion. I first consider a version of the model in which the shocks have no persistence and illustrate key features of the two targeting frameworks. I then introduce persistence in the shocks. I also study a range of alternate calibrations, to test the robustness of the findings.

\textsuperscript{10}As a consequence, the stylized model tends to understate the frequency and duration of ZLB episodes. The model predicts the policy rate hits the ZLB about 3 percent of the time and the expected duration of a ZLB episode is 3 quarters (not shown). In actuality, the federal funds rate has been near the ZLB since the end of 2008.
3.1 Purely temporary shocks

I start the evaluation by assuming the shocks have no persistence, setting $\rho_{y,u,v}$ to zero in the baseline calibration. Figure 1 shows the expected evolution of the economy after each of the three types of shocks considered in the model.\(^{11}\) Shown is the response of the real interest rate, price level, and nominal GDP level.

The top panel shows the response to a positive technology shock, while the middle panel shows the response to a negative mark-up shock. Both types of supply shock put downward pressure on prices in the model, but the outcome depends on the targeting framework. With nominal GDP level targeting (dashed green lines), the real interest rate is nearly unchanged, prices fall, and nominal GDP is stable. But with strict price level targeting (solid blue lines), the real interest rate falls, nominal GDP rises, and prices are stable.\(^{12}\) Basically, each targeting framework is able to fully achieve its intended goal in response to supply shocks. However, to achieve the goal, a nominal GDP level target requires the burden of the shocks to be shared by prices and output, while a strict price level target requires the entire burden of the shocks to be transferred onto output (not shown).

The bottom panel of the figure shows the response to a negative demand shock, which exerts downward pressure on output and prices in the model. Given the size of the shock, the central bank cuts the nominal policy rate (not shown) all the way to the ZLB. In both targeting frameworks, during the ZLB episode, the real interest rate falls, prices fall, and nominal GDP falls. However, with strict price level targeting the real interest rate stays for a longer time below its equilibrium value, which implies a greater degree of monetary policy stimulus and therefore a smaller downturn in the economy. The reason for this better performance is that, as noted earlier, a strict price level target implies a greater dependence of current policy decisions on past policy actions, and thus a surge in economic activity and inflation after the ZLB episode. By contrast, during the ZLB episode, nominal GDP level targeting provides less

\(^{11}\)Shown are expected paths after 3 standard deviation shocks. The expected paths are obtained by averaging across 10,000 stochastic simulations.

\(^{12}\)In response to the mark-up shock, the nominal policy rate is lowered substantially but it does not reach the ZLB with this calibration (figure 1).
policy stimulus and, ironically, leads to a larger fall in nominal GDP.

[Figure 1 about here]

To rank the two targeting frameworks, table 3 summarizes their performance. The table reports the expected frequency and duration of ZLB episodes, as well as the welfare loss due to business cycles.\textsuperscript{13} Each line shows the outcome for one type of shock only. The top panel shows the results for strict price level targeting, while the bottom panel shows the outcome for nominal GDP level targeting. Regarding technology shocks, strict price level targeting is the more effective framework because it neutralizes the effects of the shock entirely and the total welfare loss is zero, while nominal GDP level targeting fails to insulate the economy and causes a welfare loss. Regarding mark-up shocks, strict price level targeting is the less effective framework because to offset inflationary pressures it causes relatively costly fluctuations in output, while nominal GDP level targeting improves the inflation-output tradeoff faced by the central bank and causes a smaller total welfare loss. And regarding demand shocks, the two targeting frameworks are equally effective in dealing with the ZLB and the related tradeoff between current and future output.\textsuperscript{14} However, as shown in the next section, the policy ranking depends on the persistence of the shocks.

[Table 3 about here]

3.2 Persistent shocks

I now introduce persistence in the shocks and use the baseline calibration. Figure 2 shows the expected evolution of the model economy with persistent shocks.\textsuperscript{15} As shown in the top and middle panels, even though the shocks are persistent, each targeting framework still fully achieves its intended goal in response to technology and mark-up shocks. Regarding the

\textsuperscript{13}To calculate the welfare loss, first the value of objective function (3) is obtained by averaging across 10,000 stochastic simulations each 1,000 periods long after a burn-in period. This value is then converted into a permanent consumption loss, as explained in Appendix A.3.

\textsuperscript{14}Under both frameworks, due to demand shocks, the policy rate hits the ZLB about 8 percent of the time and the expected duration of a ZLB episode is 1 quarter (table 3).

\textsuperscript{15}Again shown are expected paths after 3 standard deviation shocks.
response to demand shocks that push the economy into a ZLB episode, as shown in the bottom panel, the real interest rate still falls by more under a strict price level target, which implies a greater degree of monetary policy stimulus to the economy. As a result, if the analysis takes into account shock persistence, the fall in nominal GDP during a ZLB episode is substantially larger under a nominal GDP level target.

Figure 2 about here

Next, table 4 summarizes the performance of the two targeting frameworks with persistent shocks. It shows that, even if technology shocks are persistent, strict price level targeting is still the more effective framework because it neutralizes such shocks entirely, while under nominal GDP level targeting economic performance deteriorates. If mark-up shocks are persistent, the two frameworks are now equally effective in terms of overall welfare. The reason is that persistent mark-up shocks give rise to costly fluctuations in prices under nominal GDP level targeting, which are comparable in size to the costs of output volatility under strict price level targeting. And if demand shocks are persistent, strict price level targeting is now the more effective framework.\(^\text{16}\) This occurs because persistent demand shocks cause costly fluctuations in both prices and output under nominal GDP level targeting. In sum, for all types of shocks considered, introducing shock persistence in the analysis leads to a deterioration in economic performance under nominal GDP level targeting.

Table 4 about here

3.3 Alternate calibrations

As the next step in the analysis, I consider a number of deviations from the baseline calibration of the model economy. For each change, the model implied parameters (β, κ and λ) are

\(^{16}\)ZLB episodes with persistent demand shocks are more frequent and last longer under strict price level targeting. The frequency of ZLB episodes is 17 percent of the time under strict price level targeting, but falls to 11 percent under nominal GDP level targeting; at the same time, the expected duration of a ZLB episode falls from 3 to 2 quarters (table 4).
adjusted accordingly. Table 5 summarizes the performance of the two targeting frameworks with the parameters changes.\footnote{Changes to $\omega$ are not reported, because varying $\omega$ as much as ±50 percent makes no noticeable difference for the implied parameters and results.}

I start with changes on the demand side of the economy. First, the equilibrium rate of interest is lowered substantially ($\beta = 0.993$), which implies that monetary policy is now more severely constrained by the ZLB.\footnote{The equilibrium interest rate is lowered from 4 to 3 percent annual. Thus, the frequency and duration of ZLB episodes rise under both targeting frameworks (table 5).} As a consequence, inflation and output volatility rise under both targeting frameworks relative to the baseline, but the welfare loss increases by more under nominal GDP level targeting. This occurs because, as noted earlier, such a policy framework is less effective in dealing with demand shocks that push the economy into a ZLB episode.\footnote{In the absence of the ZLB constraint, demand shocks are countered entirely by changes in the nominal interest rate. However, if technology and mark-up shocks are persistent, strict price level targeting is still the more effective framework in terms of overall welfare (table 4).} Second, the interest elasticity of real aggregate demand is lowered substantially ($\varphi = 1$), which entails that monetary policy is now more effective at stabilizing economic activity, as changes in the nominal interest rate have larger effects on output.\footnote{At the same time, the supply side of the economy is also affected. As $\kappa$ rises, changes in output have larger effects on prices.} On net, inflation and output volatility fall under both targeting frameworks relative to the baseline, but strict price level targeting is still the more effective framework in terms of overall welfare.

[Table 5 about here]

I now consider a range of parameter values on the supply side of the model economy.\footnote{In the baseline calibration, the duration between price changes is 3 quarters ($\alpha = 0.66$) and the desired markup is 15 percent ($\theta = 7.66$). In the alternate calibrations reported in table 5, the price duration ranges from 2 to 4 quarters ($\alpha = 0.5$ and 0.75, respectively) and the markup ranges from 11 to 25 percent ($\theta = 10$ and 5, respectively).} In the model, if firms change prices less frequently or face more competition ($\alpha = 0.75$ and $\theta = 10$, respectively), changes in output have larger effects on prices ($\kappa$ rises). As a result, as table 5 shows, inflation and output volatility generally rise under both targeting frameworks relative to the baseline, but the welfare loss increases by more under nominal GDP level targeting. Conversely, if firms change prices more frequently or face less competition ($\alpha = 0.5$ and $\theta = 5$, respectively), inflation and output volatility generally fall under both frameworks relative to the baseline.
the baseline, but strict price level targeting is still the more effective framework in terms of overall welfare. Finally, I examine the persistence of the mark-up shocks. As shown, if mark-up shocks have no persistence \( (\rho_u = 0) \), inflation falls under nominal GDP level targeting, but not enough to change the policy ranking. Overall, the ranking of the two targeting frameworks with persistent shocks is robust to a wide range of alternate calibrations.

4 Conclusion

Shedding light on recent proposals directed at major central banks to adopt a nominal GDP level target, this article compares nominal GDP level targeting to strict price level targeting in a standard model often used for monetary policy analysis. In the model, the central bank operates under optimal discretion and faces a ZLB constraint, and the economy is buffeted by supply and demand shocks. The stylized model is calibrated to recent U.S. data and offers a clear illustration of the tradeoffs faced by the central bank. The two targeting frameworks are ranked in terms of performance, based on the model’s social welfare function.

The analysis suggests that, if the economy is only buffeted by purely temporary shocks to inflation, nominal GDP level targeting may be preferable because it requires the burden of the shocks to be shared by prices and output, while strict price level targeting causes costly fluctuations in output. But in the presence of persistent supply and demand shocks, strict price level targeting may be superior because it induces greater policy inertia and improves the tradeoffs faced by the central bank. During ZLB episodes, ironically, nominal GDP level targeting leads to larger falls in nominal GDP. Such results are shown to be robust to a wide range of alternate calibrations. Still, as the analysis is conducted in a stylized model, further study is needed to extend the results to a broader class of models.
A Appendix

A.1 Equilibrium conditions

I first derive the equilibrium conditions and then summarize them in a table.

**Strict price level targeting.** The problem can be written as

\[
V(s_t) = \max \left[ - (p_{t-1} + \pi_t)^2 + \beta E_t V(s_{t+1}) \right]
\]

subject to (1), (2) and \( i_t \geq 0 \)

and \( E_t y_{t+1} \) given

Write the period Lagrangian

\[
L_t = - (p_{t-1} + \pi_t)^2 + \beta E_t V(s_{t+1})
+ m_{1t} \left[ \pi_t - \beta E_t \pi_{t+1} - \kappa (y_t - y_t^n) - u_t \right]
+ m_{2t} \left[ - y_t + E_t y_{t+1} - \varphi (i_t - r - E_t \pi_{t+1} - v_t) \right]
\]

and \( E_t y_{t+1} \) given

The Kuhn-Tucker conditions are

\[
\frac{\partial L_t}{\partial \pi_t} = -2 (p_{t-1} + \pi_t) + m_{1t} = 0 \quad (5)
\]

\[
\frac{\partial L_t}{\partial y_t} = -2 (p_{t-1} + \pi_t) - \kappa m_{1t} - m_{2t} = 0 \quad (6)
\]

\[
\frac{\partial L_t}{\partial i_t} \cdot i_t = -\varphi m_{2t} \cdot i_t = 0, \quad m_{2t} \geq 0, \quad i_t \geq 0 \quad (7)
\]

**Nominal GDP level targeting.** Similarly the problem can be written as
\[ V(s_t) = \max \left[ - (p_{t-1} + \pi_t + y_t)^2 + \beta E_t V(s_{t+1}) \right] \]

subject to (1), (2) and \( i_t \geq 0 \)

and \( E_t y_{t+1} \) given

The Kuhn-Tucker conditions then are

\[ \frac{\partial L_t}{\partial \pi_t} = -2 (p_{t-1} + \pi_t + y_t) + m_{1t} = 0 \quad (8) \]
\[ \frac{\partial L_t}{\partial y_t} = -2 (p_{t-1} + \pi_t + y_t) - \kappa m_{1t} - m_{2t} = 0 \quad (9) \]
\[ \frac{\partial L_t}{\partial i_t} \cdot i_t = \varphi m_{2t} \cdot i_t = 0, \quad m_{2t} \geq 0, \quad i_t \geq 0 \quad (10) \]

The equilibrium conditions are summarized in the following table:

<table>
<thead>
<tr>
<th>Policy framework</th>
<th>Equilibrium conditions</th>
<th>State vector ( s_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple policy rule</td>
<td>(1), (2) and (4)</td>
<td>((y^n_t, u_t, v_t, \hat{i}^n_t_{t-1}))</td>
</tr>
<tr>
<td>Strict price level targeting</td>
<td>(1), (2) and (5)-(7)</td>
<td>((y^n_t, u_t, v_t, p_{t-1}))</td>
</tr>
<tr>
<td>Nominal GDP level targeting</td>
<td>(1), (2) and (8)-(10)</td>
<td>((y^n_t, u_t, v_t, p_{t-1}))</td>
</tr>
</tbody>
</table>

### A.2 Numerical procedure

I find a numerical solution, as in Billi (2011), as a fixed-point in the equilibrium conditions. To do so, the state vector is discretized into a grid of interpolation nodes, with a support of ±4 standard deviations for each state variable which is large enough to avoid erroneous extrapolation. If the state is not on this grid, the response function is evaluated with multilinear interpolation. The approximation residuals are evaluated at a finer grid, to ensure the accuracy of the results. The expectations function is evaluated with Gaussian-Hermite quadrature. The initial guess is the linearized solution that ignores the ZLB constraint.
A.3 Permanent Consumption Loss

I obtain the permanent consumption loss as in Billi (2011). The expected lifetime utility of the representative household is validly approximated by

\[ E_0 \sum_{t=0}^{\infty} \beta^t U_t = \frac{U_cC}{2} \frac{\alpha\theta (1 + \omega\theta)}{(1 - \alpha)(1 - \alpha\beta)} L, \] (11)

where \( C \) is steady-state consumption; \( U_c > 0 \) is steady-state marginal utility of consumption; and \( L \geq 0 \) is the value of objective function (3).

At the same time, a steady-state consumption loss of \( \mu \geq 0 \) causes a utility loss

\[ E_0 \sum_{t=0}^{\infty} \beta^t U_t C\mu = \frac{1}{1 - \beta} U_cC\mu. \] (12)

Equating the right sides of (11) and (12) gives

\[ \mu = \frac{1 - \beta}{2} \frac{\alpha\theta (1 + \omega\theta)}{(1 - \alpha)(1 - \alpha\beta)} L. \]

References


Table 1: Baseline calibration of the model

<table>
<thead>
<tr>
<th>Definition</th>
<th>Parameter</th>
<th>Numerical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Interest elasticity of aggregate demand</td>
<td>$\varphi$</td>
<td>6.25</td>
</tr>
<tr>
<td>Share of firms keeping prices fixed</td>
<td>$\alpha$</td>
<td>0.66</td>
</tr>
<tr>
<td>Price elasticity of demand</td>
<td>$\theta$</td>
<td>7.66</td>
</tr>
<tr>
<td>Elasticity of a firms’ marginal cost</td>
<td>$\omega$</td>
<td>0.47</td>
</tr>
<tr>
<td>Slope of aggregate supply curve</td>
<td>$\kappa$</td>
<td>0.024</td>
</tr>
<tr>
<td>Weight on output gap</td>
<td>$\lambda$</td>
<td>0.003</td>
</tr>
<tr>
<td>Std. deviation of technology shock</td>
<td>$\sigma_y$</td>
<td>0.80 percent</td>
</tr>
<tr>
<td>Std. deviation of mark-up shock</td>
<td>$\sigma_u$</td>
<td>0.05 percent</td>
</tr>
<tr>
<td>Std. deviation of demand shock</td>
<td>$\sigma_v$</td>
<td>0.80 percent</td>
</tr>
<tr>
<td>AR(1) parameter of shocks</td>
<td>$\rho_{y,u,v}$</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Note: Because in the model a period is one quarter, shown are parameter values corresponding to inflation and interest rates measured at a quarterly rate.
Table 2: Fitting the model to the data

<table>
<thead>
<tr>
<th></th>
<th>Std. deviation (pa)</th>
<th>Autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\pi$</td>
<td>$y$</td>
</tr>
<tr>
<td>U.S. data$^a$</td>
<td>1.04</td>
<td>4.01</td>
</tr>
<tr>
<td>Model results$^b$</td>
<td>0.98</td>
<td>4.82</td>
</tr>
</tbody>
</table>

a. The sample period is 1984Q1 to 2014Q4.

b. Baseline calibration, and policy follows the inertial Taylor rule (4).
Table 3: Economic performance, with purely temporary shocks

<table>
<thead>
<tr>
<th>ZLB episodes</th>
<th>Welfare lossc</th>
</tr>
</thead>
<tbody>
<tr>
<td>freq.a</td>
<td>durationb</td>
</tr>
<tr>
<td>Strict price level targeting:</td>
<td></td>
</tr>
<tr>
<td>Technology shock only</td>
<td>0</td>
</tr>
<tr>
<td>Mark-up shock only</td>
<td>0</td>
</tr>
<tr>
<td>Demand shock only</td>
<td>8</td>
</tr>
<tr>
<td>Nominal GDP level targeting:</td>
<td></td>
</tr>
<tr>
<td>Technology shock only</td>
<td>0</td>
</tr>
<tr>
<td>Mark-up shock only</td>
<td>0</td>
</tr>
<tr>
<td>Demand shock only</td>
<td>8</td>
</tr>
</tbody>
</table>

a. Expected percent of time at the ZLB

b. Expected number of consecutive quarters at the ZLB

c. Permanent consumption loss (basis points)
Table 4: Economic performance, with persistent shocks

| ZLB episodes | Welfare loss
c | freq.\(^a\) | duration\(^b\) | \(\pi\) | \(x\) | Tot. |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strict price level targeting:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology shock only</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Mark-up shock only</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Demand shock only</td>
<td>17</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td><strong>Nominal GDP level targeting:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology shock only</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Mark-up shock only</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Demand shock only</td>
<td>11</td>
<td>2</td>
<td>7</td>
<td>10</td>
<td>17</td>
<td></td>
</tr>
</tbody>
</table>

a. Expected percent of time at the ZLB
b. Expected number of consecutive quarters at the ZLB
c. Permanent consumption loss (basis points)
Table 5: Economic performance, alternate calibrations

<table>
<thead>
<tr>
<th>Strict price level targeting:</th>
<th>ZLB episodes</th>
<th>Welfare loss$^c$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>freq.$^a$</td>
<td>duration$^b$</td>
</tr>
<tr>
<td>Baseline</td>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>Lower steady-state real rate ($\beta = 0.993$)</td>
<td>27</td>
<td>5</td>
</tr>
<tr>
<td>Smaller demand elasticity ($\varphi = 1$)</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>Prices less sticky ($\alpha = 0.5$)</td>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>Prices more sticky ($\alpha = 0.75$)</td>
<td>13</td>
<td>3</td>
</tr>
<tr>
<td>Less competition ($\theta = 5$)</td>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>More competition ($\theta = 10$)</td>
<td>14</td>
<td>3</td>
</tr>
<tr>
<td>Purely temporary mark-up shocks ($\rho_u = 0$)</td>
<td>16</td>
<td>3</td>
</tr>
<tr>
<td>Nominal GDP level targeting:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>Lower steady-state real rate ($\beta = 0.993$)</td>
<td>17</td>
<td>3</td>
</tr>
<tr>
<td>Smaller demand elasticity ($\varphi = 1$)</td>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>Prices less sticky ($\alpha = 0.5$)</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>Prices more sticky ($\alpha = 0.75$)</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>Less competition ($\theta = 5$)</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>More competition ($\theta = 10$)</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>Purely temporary mark-up shocks ($\rho_u = 0$)</td>
<td>10</td>
<td>2</td>
</tr>
</tbody>
</table>

a. Expected percent of time at the ZLB  
b. Expected number of consecutive quarters at the ZLB  
c. Permanent consumption loss (basis points)
Figure 1: Evolution of the economy, with purely temporary shocks

Note: Shown are the expected paths after 3 standard deviation shocks, for each type of shock as in table 3.
Figure 2: Evolution of the economy, with persistent shocks

Note: Shown are the expected paths after 3 standard deviation shocks, for each type of shock as in table 4.