Optimal Inflation for the U.S. Economy[1]

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Abstract

Previous studies do not provide direct estimates of the optimal (long-run mean) inflation rate that maximizes the public's economic well-being, which is essential information for formulating a long-run inflation target. This paper provides direct estimates of the optimal inflation rate in a small New-Keynesian model subject to an occasionally-binding zero lower bound on nominal interest rates and worst-case scenarios of model uncertainty. The optimal inflation rate is between 0.7 percent per year (no model uncertainty) and 1.4 percent per year (extreme model uncertainty) when measured using the PCE price index. The paper shows that the policymaker can practically implement the optimal inflation rate and completely avoid hitting the zero lower bound when it commits merely to a superinertial Taylor rule with a sufficiently high degree of policy inertia. The optimal inflation rate, however, is more than 14 percent per year when the policymaker makes no commitment about future policies.

Keywords: commitment, discretion, liquidity trap, long-run tradeoffs, long-run stationary distribution, nonlinear monetary policy, stochastic robust control

JEL Classification: C63, E31, E52

1 Introduction

Economists, policymakers, and the public agree that high inflation must be avoided because it ultimately reduces standards of living.[1] For a number of reasons, however, inflation can be too low. Thus, central banks typically have a long-run inflation goal that is low, but above zero.

1.1 The Case for Low but Positive Inflation Goals

One reason why inflation can be too low stems from the *zero lower bound on nominal interest rates*.[2] Nominal interest rates are low when inflation is expected to remain low. Since central banks typically lower short-term interest rates to counteract slowing economic activity, very low inflation limits the extent central banks can ease policy in response to an economic slowdown. Thus, policymakers and several economists argue in favor of low, positive rates of inflation to leave enough room to ease policy in response to an economic slowdown.[3,4]

There are a number of other reasons why inflation can be too low:

Measurement bias in inflation. Available measures of inflation tend to be biased upward. Recent estimates place the measurement bias for the personal consumption expenditure (PCE) price index, the Federal Reserve's preferred measure of inflation, at about 0.5 percentage point per year.[5]

Downward wage rigidity. Tobin (1972) argues that nominal wages may be downwardly rigid if firms are unable to make nominal wage cuts because workers are unwilling to accept them. Thus, a little inflation may make it easier for firms to lower real wages and maintain employment in response to declining demand.[6]

Debt–Deflation. Fisher (1933) argues that a negative inflation rate—deflation—may be a more serious problem than inflation because deflation causes a decreases in the value of collateral used to secure a loan, or another form of debt. Sustained deflation could create a vicious cycle of financial distress for banks and other lenders, which would lead to more downward pressure on prices. Thus, a little inflation may be desirable to insure against debt-deflation.[7]

In practice, the zero lower bound and measurement bias in inflation are the two main reasons why central banks typically have a long-run inflation goal above zero. At the same time, central banks simultaneously insure against downward wage rigidity and debt-deflation when the inflation goal is above zero.

[1] Fischer (1996) discusses the costs of inflation.

[2] In theory, achieving negative nominal interest rates is feasible levying a tax on money holdings or giving up free convertibility of financial assets into cash. Buiter and Panigirtzoglou (2003), and Goodfriend (2000) discuss this idea.

[3] For example, Vickrey (1954), Phelps (1972), Okun (1981), Summers (1991), Fischer (1996), Fuhrer and Madigan (1997), Krugman (1998), McCallum (2000), Orphanides and Wieland (2000), Svensson (2003), Bernanke, Reinhart and Sack (2004), Kato and Nishiyama (2005), and Wolman (2005).

[4] However, Friedman (1969) argues that the opportunity cost of holding money is zero when nominal interest rates are zero. Nominal interest rate are equal to real interest rates plus expected inflation (the Fisher identity). Since real interest rates are usually positive, inflation is expected to be negative when nominal interest rates are zero.

[5] Former Federal Reserve Governor Gramlich (2003) discusses the measurement bias in inflation.

[6] Akerlof and Dickens (2007) argue whether downward wage rigidity is relevant for monetary policy.

[7] Bernanke (2004) studies the debt-deflation problem in the Great Depression.

1.2 Contribution

This paper provides the first estimates of the optimal (long-run mean) inflation rate in a small New-Keynesian sticky-price model subject to an occasionally-binding zero lower bound on nominal interest rates and worst-case scenarios of model uncertainty. The model is calibrated to recent U.S. data.

This is the first paper to study robustly optimal monetary policy considering the zero lower bound. As such, it unifies two separate strands of economics literature. On the one hand, as Sims (2001) points out, studies of monetary policy and model uncertainty do not consider the zero lower bound.[8] On the other hand, studies of monetary policy and the zero lower bound do not consider model uncertainty.[9]

The paper makes also an important technical contribution, because the model is significantly more difficult to solve than in earlier studies. The model has five continuos state variables, which is unusually high and problematic with an occasionally-binding constraint. Thus, despite the use of a state-of-the-art computing environment, available numerical procedure are not suitable.[10]

Table 1 summarizes the estimates:

First, the optimal inflation rate is between 0.7 percent per year (no model uncertainty) and 1.4 percent per year (extreme model uncertainty) when measured using the PCE price index.[11] At the same time, the federal funds rate hits the zero lower bound between about 4 percent of the time (no model uncertainty) and 7 percent of the time (extreme model uncertainty), and stays there for only about two consecutive quarters.[12]

Second, the optimal inflation rate is almost 9 percent per year when the policymaker commits merely to a non-inertial Taylor rule, rather than a completely specified policy plan. At the same time, the policymaker can completely avoid hitting the zero lower bound when inflation is that high.

Third, the policymaker can practically implement the optimal inflation rate, maximize the public's economic well-being, and completely avoid hitting the zero lower bound when it commits merely to a superinertial Taylor rule with a sufficiently high degree of policy inertia.[13]

[8] For example, Giannoni (2002), Onatski and Stock (2002), Onatski and Williams (2003), Giordani and Söderlind (2004), Walsh (2004), Woodford (2005), Cateau (2007), Dennis (2007), and Hansen and Sargent (2008).

[9] Eggertsson and Woodford (2003), and Jung, Teranishi and Watanabe (2005) study optimal monetary policy in a small New-Keynesian. Both papers assume a stochastic process for the shocks with an absorbing state or rely on solution methods that impose certainty equivalence. Thus, the optimal (long-run mean) inflation rate cannot be determined with such approach. Adam and Billi (2006, 2007) characterize the optimal policy for a more general stochastic process like the one studied here, but do not consider model uncertainty.

[10] The numerical procedure of Billi (2005) and Adam and Billi (2006, 2007) are based on the value function approach and use high-order polynomials to obtain a 'smooth' approximation of the value function. The numerical procedure in this paper is based instead on the Euler-equation approach, which bypasses the calculation of the value function's Jacobian and is less prone to memory limitations, and uses a piecewise-linear to more accurately approximate the 'kinks' in the response function. Indeed, any numerical procedure designed for solving models with smooth response functions, as for example that of Krueger and Kubler (2004), is not suitable for occasionally-binding constraints.

[11] In this context, extreme model uncertainty is the greatest uncertainty surrounding the model's parameters for which inflation expectations remain anchored and the numerical procedure can identify an equilibrium.

[12] Intuitively, if there is greater uncertainty surrounding the parameters of the model, the uncertainty about the response of the economy to shocks increases. This uncertainty about the structure and response of the economy leads to uncertainty about the effects of monetary policy. Thus, higher inflation is required to avoid hitting the zero lower bound too often when there is model uncertainty.

[13] The relatively high weight on last period's nominal interest rate level in the superinertial Taylor rule does not entail

	Commitment No Model Uncertainty[a]	Commitment With Model Uncertainty[b]	Non-inertial Taylor Rule[c]	Superinertial Taylor Rule[d]	Discretion No Model Uncertainty[e]
Optimal inflation rate:					
Without measurement bias	0.2	0.9	8.0	0.5	13.4
With 0.5% bias (PCE Price Index)	0.7	1.4	8.5	1.0	13.9
Percent of time funds rate is zero	3.7	7.0	0.0	0.0	0.0
Number of quarters funds rate is zero	1.8	2.4	0.0	0.0	0.0
Standard deviation of:					
Output Gap	1.2	1.6	0.9	1.7	1.9
Inflation	1.9	2.9	1.9	1.3	2.3
Federal funds rate	2.4	3.2	2.6	0.8	3.3

Notes: Results are in annualized percentage points.
[a] The policymaker commits to an equilibrium path that maximizes social welfare subject to a zero lower bound on nominal interest rates, without model uncertainty.
[b] The policymaker commits to an equilibrium path, while facing the greatest model uncertainty for which inflation expectations remain anchored.
[c] The policymaker commits to set the nominal interest rate with 1.5-to-1 response to inflation deviations from target and 0.5-to-1 response to output gap deviations, without model uncertainty.
[d] The policymaker commits to set the nominal interest rate with in addition 5-to-1 weight on last period's nominal interest rate level, without model uncertainty.
[e] The policymaker re-optimizes every period, without commitment or model uncertainty.

Table 1: Optimal Inflation in the Small New-Keynesian Model

Fourth, the optimal inflation rate is as high as 14 percent per year when the policymaker makes no commitment about future policies or re-optimizes every period (no model uncertainty). The optimal inflation rate is even higher when model uncertainty is considered.[14]

1.3 Previous Studies of Inflation Goals and the Zero Lower Bound

Previous studies estimate the tradeoffs between the long-run inflation goal, the frequency of zero nominal interest rates, and macroeconomic performance.

For example, Reifschneider and Williams (2000) simulate the FRB/US model, which is a large-scale structural model the Federal Reserve Board uses for forecasting and policy analysis.[15] Table 2 shows the estimates of the tradeoffs when the policymaker commits to a non-inertial Taylor rule. If the long-run inflation goal is zero percent, the federal funds rate hits the zero lower bound 14 percent of the time and stays there for six consecutive quarters.

an extremely high degree of nominal interest rate inertia. The autocorrelation of the nominal interest rate is 0.85, which is between that with commitment (0.80) and discretion (0.87) when there is no model uncertainty.

[14] Since the optimal inflation rate appears implausibly high when the policymaker makes no commitment, the paper focuses on the study of optimal inflation when the policymaker commits to an equilibrium path or a Taylor rule.

[15] As most macro models, the FRB/US model does not incorporate downward wage rigidity or a debt-deflation channel.

	Long-Run Inflation Goal				
	0	1	2	3	4
Percent of time federal funds rate is zero	14	9	5	1	< 1
Number of consecutive quarters funds rate is zero	6	5	4	3	2
Standard deviation of:					
Output Gap (CBO estimate)	3.6	3.2	3.0	2.9	2.9
Inflation (PCE Price Index)	2.0	1.9	1.9	1.9	1.9
Federal funds rate	2.3	2.4	2.5	2.5	2.5

Notes: The policymaker commits to a non-inertial Taylor rule which sets the nominal interest rate level with 1.5-to-1 response to inflation deviations from target and 0.5-to-1 response to output deviations subject to a zero lower bound. Results are in annualized percentage points.

Table 2: Optimal Inflation with Commitment to a Taylor Rule in the FRB/US Model

As the inflation goal is raised, however, the funds rate hits the zero lower bound less often and macroeconomic performance improves. If the inflation goal is 4 percent, the funds rate hits the zero lower bound less than 1 percent of the time and stays there for only two consecutive quarters.[16] Since the variability of both output *and* inflation fall when the inflation goal rises, higher inflation goals appear unambiguously better. Thus, the optimal inflation rate cannot be determined with such approach. Other studies provide similar estimates.[17]

The remainder of this paper is structured as follows. Section 2 explains the model, then section 3 explains the equilibrium definition when the policymaker commits to an equilibrium path. In section 4, the model is calibrated to recent U.S. data. Section 5 shows that uncertainty about the future state of the economy raises the optimal (long-run mean) inflation rate. Section 6 shows that greater model uncertainty also raises the optimal inflation rate. Section 7 shows the robustness of the estimates of the optimal inflation rate when the policymaker commits merely to a Taylor rule. Section 8 briefly concludes.

2 Commitment to an Equilibrium Path

The setting adopts the well-known sticky-price version of the small New-Keynesian model, which is discussed in-depth by Clarida, Galí and Gertler (1999), Woodford (2003a), Galí (2008), and others.[18] The private sector consists of a representative consumer and firms in monopolistic competition facing restrictions on the frequency of price adjustments à la Calvo (1983). The policymaker commits to the objective of maximizing welfare for the consumer.

The setting also adopts the robust control approach of Hansen and Sargent (2008) to study optimal

[16] Reifschneider and Williams show also that incorporating policy inertia in the Taylor rule changes the estimates of the tradeoffs and may lower the frequency of zero nominal interest rates.

[17] Coenen, Orphanides, and Wieland (2004) were among the first to estimate the tradeoffs in a structural model of the U.S. economy. Their model has less sectoral detail than the FRB/US model, but shares the same basic features.

[18] To save space, the complete derivation of the small New-Keynesian model is not shown here.

policy in worst-case scenarios of model uncertainty. Since the policymaker maximizes private-sector welfare, it has the same model of the economy as the private sector. The policymaker is concerned about model uncertainty, because model uncertainty hinders the policymaker's ability to predict the evolution of the economy and shape private-sector expectations.[19] The model is viewed as an approximation of the true model of the economy which is unknown to the policymaker, but is known to be in a neighborhood around its approximating model.[20]

Thus, the optimal policy problem using the robust control approach is

$$\max_{\{\pi_t, x_t, i_t\}} \min_{\{w_{1t}, w_{2t}\}} -\hat{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\pi_t - \gamma\pi_{t-1}\right)^2 + \lambda x_t^2 - \Theta\left(w_{1t}^2 + w_{2t}^2\right) \right] \tag{1}$$

s.t.

$$\pi_t - \gamma\pi_{t-1} = \beta \hat{E}_t \left(\pi_{t+1} - \gamma\pi_t\right) + \kappa x_t + u_t \tag{2}$$

$$x_t = \hat{E}_t x_{t+1} - \varphi\left(i_t - \hat{E}_t \pi_{t+1} - r_t^n\right) \tag{3}$$

$$u_t = \rho_u u_{t-1} + \sigma_{\varepsilon u}\left(\varepsilon_{ut} + w_{1t}\right) \tag{4}$$

$$r_t^n = \left(1 - \rho_r\right) r_{ss} + \rho_r r_{t-1}^n + \sigma_{\varepsilon r}\left(\varepsilon_{rt} + w_{2t}\right) \tag{5}$$

$$i_t \geq 0 \tag{6}$$

where $\hat{E}_t$ denotes the expectations operator conditional on information available at time t. The accent is added above the expectations operator to indicate that expectations are formed in a worst-case scenario of model uncertainty. π_t is the inflation rate, and x_t is the output gap or the deviation of output from its flexible-price equilibrium.[21] And i_t is the nominal interest rate.[22]

The small New-Keynesian model is developed from explicit micro-foundations. As a result, the policymaker's objective function can be derived taking a second-order Taylor series approximation to the expected life-time utility of the consumer. The resulting welfare-theoretic objective function (1) of the policymaker is quadratic in deviations of output from the socially efficient level and deviations of the unanticipated component of inflation from zero.[23] $\beta \in (0, 1)$ is the subjective discount factor. The weight assigned to the goal of output stability, relative to price stability,

[19] In a more general setting the private sector may also face model uncertainty, perhaps to a different extent than the policymaker.

[20] The setting is as a social-planning game, played between the policymaker and a fictitious adversary agent or nature. The policymaker selects the equilibrium paths of inflation, the output gap, and the nominal interest rate $\{\pi_t, x_t, i_t\}_{t=0}^{\infty}$ to achieve time-zero optimal policy that maximizes welfare for the consumer. At the same time, nature selects worst-case shocks $\{w_{1t}, w_{2t}\}_{t=0}^{\infty}$ to distort the model and minimize welfare. Thus, the policymaker selects optimal policy that is robust to such worst-case shocks.

[21] Output is efficient at its deterministic steady state level due to an output subsidy that neutralizes the distortions from monopolistic competition.

[22] Abstracting from money-demand distortions associated with positive nominal interest rates, the model can be interpreted as the 'cashless limit' of a model with money holdings.

[23] The unanticipated component of price changes $(\pi_t - \gamma\pi_{t-1})$, not the anticipated component from indexation $(\gamma\pi_{t-1})$, matters for social welfare.

$$\lambda \equiv \frac{\kappa}{\theta} > 0 \tag{7}$$

is a function of the structure of the model economy. $\theta > 1$ is the price elasticity of demand substitution among differentiated goods produced by firms operating in monopolistic competition.

Equation (2) is a log-linear approximation to the aggregate-supply curve, which describes the optimal price-setting behavior of firms under staggered price setting. The slope parameter

$$\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \frac{\varphi^{-1}+\omega}{1+\omega\theta} > 0 \tag{8}$$

is a function of the structure of the model economy. $\omega > 0$ is the elasticity of a firm's real marginal cost with respect to its own output level. Each period, a share $\alpha \in (0,1)$ of randomly picked firms cannot adjust their prices and the remaining $(1-\alpha)$ firms get to choose prices optimally. Prices that are not optimized are indexed to the most recent aggregate price index, and $\gamma \in [0,1)$ is the degree of indexation. The shifter of the aggregate-supply curve u_t is interpreted as a 'mark-up' shock or the variation over time in the degree of monopolistic competition between firms.

Equation (3) is a log-linear approximation to the intertemporal Euler equation describing the representative consumer's private expenditure decisions. $\varphi > 0$ is the intertemporal elasticity of substitution or the real-rate elasticity of the output gap. Shifting the Euler equation is the 'natural' real-rate of interest shock r_t^n.[24]

Equations (4) and (5) describe the evolution of the exogenous mark-up shock u_t and the real-rate shock r_t^n. The shocks are AR(1) stochastic processes with autoregressive coefficients $\rho_j \in (-1,1)$ for $j = u, r$. The deterministic steady state real interest rate is $r_{ss} \equiv 1/\beta - 1$, such that $r_{ss} \in (0,+\infty)$. The innovations $\sigma_{\varepsilon j}\varepsilon_{jt}$ are independent across time and cross-sectionally, and normally distributed with mean zero and standard deviations $\sigma_{\varepsilon j} \geq 0$ for $j = u, r$.

The worst-case shocks w_{1t} and w_{2t} alter the evolution of the exogenous shock processes (4) and (5). Altering the (expected) evolution of the economy, the worst-case shocks affect directly the formation of expectations. Thus, model uncertainty limits the policymaker's ability to shape expectations. The parameter $\Theta \geq 0$ in the objective function (1) determines the degree of model uncertainty or the distance between the approximating model and a worst-case scenario. Intuitively, a more severe modeling misspecification can arise if Θ is small, but there is no model uncertainty if $\Theta \to +\infty$.

Equation (6) is the zero lower bound on the nominal interest rate. Mainly for reasons of analytical tractability, the economics literature typically abstracts from the zero bound and thereby assumes that the policymaker can achieve negative nominal interest rates.[25] The simpler problem (1)-(5) without the zero bound can be solved with standard linear-quadratic methods.

[24] The real-rate shock summarizes all shocks that under flexible prices generate variation in the real interest rate; it captures the combined effects of preference shocks, productivity shocks, and exogenous changes in government expenditure.

[25] For example Woodford (2003a) or Galí (2008), and references therein.

3 Equilibrium

The infinite-horizon Lagrangian the optimal policy problem (1)-(6) in recursive form is

$$\begin{aligned}\max_{\{\pi_t,x_t,i_t\}}\min_{\{m_{1t},m_{2t},w_{1t},w_{2t}\}} \hat{\mathcal{L}} \equiv \hat{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ & -(\pi_t - \gamma\pi_{t-1})^2 - \lambda x_t^2 + \Theta\left(w_{1t}^2 + w_{2t}^2\right) \\ & + m_{1t}\left[(1+\beta\gamma)\pi_t - \gamma\pi_{t-1} - \kappa x_t - u_t\right] - m_{1t-1}\pi_t \\ & + m_{2t}\left[-x_t - \varphi\left(i_t - r_t^n\right)\right] + m_{2t-1}\beta^{-1}\left(x_t + \varphi\pi_t\right)\Big\} \\ & \text{s.t.} \\ \text{Equations (4)-(6) for all } t \geq 0 &\end{aligned} \tag{9}$$

where m_{1t} and m_{2t} are the Lagrange multipliers for the intertemporal equilibrium conditions (2) and (3), respectively.

The Kuhn-Tucker conditions are equations (2), (3), and the marginal conditions

$$\partial\hat{\mathcal{L}}/\partial\pi_t = -2\left(\pi_t - \gamma\pi_{t-1}\right) + (1+\beta\gamma)\,m_{1t} - m_{1t-1} + \beta^{-1}\varphi m_{2t-1} = 0 \tag{10}$$

$$\partial\hat{\mathcal{L}}/\partial x_t = -2\lambda x_t - \kappa m_{1t} - m_{2t} + \beta^{-1} m_{2t-1} = 0 \tag{11}$$

$$\partial\hat{\mathcal{L}}/\partial i_t \cdot i_t = -\varphi m_{2t} \cdot i_t = 0, \quad m_{2t} \geq 0, \quad i_t \geq 0 \tag{12}$$

$$\partial\hat{\mathcal{L}}/\partial w_{1t} = 2\Theta w_{1t} - \sigma_{\varepsilon u} m_{1t} = 0 \tag{13}$$

$$\partial\hat{\mathcal{L}}/\partial w_{2t} = 2\Theta w_{2t} + \sigma_{\varepsilon r}\varphi m_{2t} = 0 \tag{14}$$

where equation (12) imposes that either the Lagrange multiplier for the Euler equation (3) or the nominal interest rate is zero, at each state and for all periods. The Euler equation is not binding ($m_{2t} = 0$) if the nominal interest rate is positive ($i_t > 0$). The Euler equation is binding ($m_{2t} > 0$) instead if the nominal interest rate reaches zero ($i_t = 0$).

Equations (2), (3), and (10)-(14) form a nonlinear system of five equations with five unknowns, which must be satisfied by optimal policy in equilibrium. Solving the system gives a seven-dimensional nonlinear equilibrium response function

$$\hat{y}\left(s_t\right) \equiv \left(\pi_t, x_t, i_t \geq 0, m_{1t}, m_{2t} \geq 0, w_{1t}, w_{2t} \leq 0\right) \subset R^7$$

which includes three natural policy variables (π_t, x_t, i_t), the Lagrange multipliers (m_{1t}, m_{2t}), and the worst-case shocks (w_{1t}, w_{2t}).[26] However, $w_{1t} \equiv w_{2t} \equiv 0$ and $\hat{y}\left(s_t\right) \subset R^5$ if there is no model uncertainty $(\Theta \to +\infty)$.[27]

[26] Equation (14), $m_{2t} \geq 0$, $\sigma_{\varepsilon r} \geq 0$, $\varphi > 0$ and $\Theta \geq 0$ imply $w_{2t} \leq 0$.

[27] Equations (13) and (14) imply $w_{1t} \equiv w_{2t} \equiv 0$ if $\Theta \to +\infty$. Thus, the nonlinear system satisfied by optimal policy in equilibrium simplifies to (2), (3), and (10)-(12) if there is no model uncertainty.

The equilibrium response function has a five-dimensional state space

$$s_t \equiv (u_t, r_t^n, \pi_{t-1}, m_{1t-1}, m_{2t-1} \geq 0) \subset R^5$$

which includes three natural state variables (u_t, r_t^n, π_{t-1}) as well as two endogenous co-state variables given by the lagged values of the Lagrange multipliers (m_{1t-1}, m_{2t-1}).

The co-state variables represent 'promises' kept from past policy commitments, which lead to deviations from purely forward-looking policy whenever their value differs from zero.[28] The worst-case shocks constrain, through the marginal conditions (13) and (14), the policymaker's ability to make commitments through the endogenous co-state variables. As a result of such constraints on the use of policy commitments, model uncertainty makes policy less effective.

The state in period $t+1$ depends on the state and equilibrium response in period t and the shock innovations that are unknown in period t,

$$s_{t+1} = g(s_t, \hat{y}(s_t), \varepsilon_{t+1}) \tag{15}$$

Associated with the equilibrium response function, the expectations function is

$$\hat{E}_t \hat{y}_{t+1} = \int \hat{y}\left(g(s_t, \hat{y}(s_t), \varepsilon_{t+1})\right) f(\varepsilon_{jt+1})\, d(\varepsilon_{t+1}) \tag{16}$$

where $f(\cdot)$ is the probability density function of the shock innovations $\varepsilon_t \equiv (\varepsilon_{ut}, \varepsilon_{rt}) \in R^2$.[29] Since the worst-case shocks enter directly the equilibrium expectations function, model uncertainty hampers the policymaker's ability to shape expectations.

The following definition of a stochastic robust control equilibrium is proposed.

Definition 1 (SRCE) *Assume $\sigma_{\varepsilon j} \geq 0$ for $j = u, r$ and $\Theta \geq 0$. A 'stochastic robust control equilibrium' of the optimal policy problem (1)-(6) with commitment to an equilibrium path is a nonlinear response function $\hat{y}(s_t)$, over the state s_t, with law of motion (15), such that the nonlinear system of equilibrium conditions (2), (3), and (10)-(14) is satisfied. The system simplifies to (2), (3), and (10)-(12) if there is no model uncertainty ($\Theta \rightarrow +\infty$).*

Importantly, the nonlinear system in definition 1 does not have a closed-form solution. A numerical procedure must be used to find a fixed-point in the space of nonlinear response functions. Since the number of state variables is unusually high for a model with an occasionally-binding constraint, the algorithm must be highly efficient. Appendix A.1 explains the numerical procedure.

[28] Woodford (2003b) argues that it is desirable for policy to respond to the co-state variables: The dependence of current policy actions on past commitments allows the policymaker to shape private-sector expectations of future policy and thereby makes policy more effective.

[29] When agents have 'perfect foresight' or predict future variables with certainty ($\sigma_{\varepsilon j} \rightarrow 0$ for $j = u, r$) the state in period $t+1$ is completely described by the state and equilibrium response in period t, $s_{t+1} = g(s_t, y(s_t))$. Thus, the expectations function (16) simplifies to $\hat{E}_t \hat{y}_{t+1} = \hat{y}\left(g(s_t, \hat{y}(s_t))\right)$ if agents have perfect foresight.

Parameter Definition	Assigned Value
Subjective discount factor	$\beta = 0.9914$
Real-rate elasticity of output gap	$\varphi = 6.25$
Share of firms keeping prices fixed	$\alpha = 0.66$
Price elasticity of demand	$\theta = 7.66$
Elasticity of firms' marginal cost	$\omega = 0.47$
Slope of the aggregate-supply curve	$\kappa = 0.024$
Weight on output gap in the utility function	$\lambda = 0.003$
Degree of inflation indexation	$\gamma = 0.9$
Deterministic steady state real interest rate	$r_{ss} = 3.5\%$ per year
s.d. real-rate shock innovation	$\sigma_{\varepsilon r} = 0.24\%$
s.d. mark-up shock innovation	$\sigma_{\varepsilon u} = 0.30\%$
AR(1)-coefficient of real-rate shock	$\rho_r = 0.8$
AR(1)-coefficient of mark-up shock	$\rho_u = 0.0$

Note: Quarterly values unless otherwise noted.

Table 3: Baseline Calibration

4 Calibration

The model is calibrated to the U.S. economy and the time period is one quarter. Table 3 shows the baseline parameter values, which are in quarters unless otherwise noted. The values for the main structural parameters ($\varphi, \alpha, \theta, \omega$, and the resulting κ, λ) are taken from tables 5.1 and 6.1 of Woodford (2003a). The degree of inflation indexation is $\gamma = 0.9$, which is consistent with the estimates of Giannoni and Woodford (2005) and Milani (2007) under rational expectations.[30]

The parameters describing the shock processes ($r_{ss}, \sigma_{\varepsilon r}, \rho_r$, and $\sigma_{\varepsilon u}, \rho_u$) are estimated over the period 1983:1-2002:4, with the same approach of Rotemberg and Woodford (1997) and Adam and Billi (2006).[31] The predictions of an unconstrained VAR in inflation, the output gap, and the federal funds rate are used to construct the expectations of inflation and the output gap.[32] The estimated expectations and the actual data are then plugged into equations (2) and (3). The equation residuals identify the historical shock processes u_t and r_t^n. Fitting AR(1) processes to the historical shocks justifies the shock processes in table 3.

The quarterly subjective discount factor is $\beta = (1 + r_{ss})^{-\frac{1}{4}} \approx 0.9914$, as implied by the estimate of the deterministic steady state real interest rate $r_{ss} = 3.5$ percent per year.

There is one additional parameter Θ that must be calibrated to study optimal policy in worst-

[30] Christiano, Eichenbaum and Evans (2005) assume full inflation indexation ($\gamma = 1$) in a model without the zero lower bound on nominal interest rates. It is easily verified that under full indexation the optimal policy problem is not well defined because it does not have a determinate steady state for inflation or the nominal interest rate. If there is full indexation, the change in inflation ($\pi_t - \pi_{t-1}$) matters in the welfare-theoretic objective function (1) and inflation is nonstationary.

[31] Adam and Billi (2006) estimate the historical shocks over the same period, but without inflation indexation ($\gamma = 0$). The mark-up shock is almost twice as variable with the empirically relevant degree of indexation ($\gamma = 0.9$).

[32] *Inflation* is measured as the continuously compounded rate of change in the GDP Chain-type Price Index, from the Bureau of Economic Analysis. The *output gap* is measured as the difference between Real GDP, from the Bureau of Economic Analysis, and Real Potential GDP, from the Congressional Budget Office. The *nominal interest rate* is measured as the mean effective federal funds rate, from the Federal Reserve Board of Governors.

case scenarios of model uncertainty. Hansen and Sargent (2008) propose a statistical theory of model selection to determine a context-specific value for Θ. They propose choosing a reasonable probability of making a detection error $p(\Theta)$ whether observed equilibrium outcomes may have originated from the approximating model with or without a worst-case shock.

When $\Theta \to +\infty$, the probability of making a detection error is 50 percent because in the limit there is no difference between the approximating model with or without a worst-case shock. When Θ is smaller, however, model misspecification is more severe and more easily detected. Appendix A.2 explains the computation of the detection error probability.[33]

5 Optimal Inflation With Commitment and No Model Uncertainty

5.1 Long-Run Stationary Distribution

Figure 1 shows the long-run stationary distribution for the baseline calibration if the policymaker is not concerned about model uncertainty.[34] The figure shows inflation (top panel), the output gap (middle panel), and the nominal interest rate (bottom panel) in probability density over annualized percentage values. The dashed-vertical lines indicate the stochastic steady state or long-run unconditional mean.

[Figure 1 about here]

The left-hand panel of figure 1 shows the distribution for the standard linear-quadratic solution of the model, which fails to consider the zero lower bound on nominal interest rates. As expected, the distribution is symmetric and normally distributed around the long-run mean.[35] The right-hand panel shows instead the distribution for the nonlinear solution of the model, which does consider the zero bound. The distribution of inflation and the output gap remains almost normal and symmetric around the mean; however, the distribution of the nominal interest rate is truncated at zero and positively skewed.[36]

Figure 2 shows the long-run stationary distribution for an alternative scenario with the shock innovations 50 percent more variable than the baseline. Although the distribution of inflation and the output gap remains almost normal and symmetric around the mean, the mean inflation rate and the mean nominal interest rate are higher if the zero bound is considered.[37]

[33] The detection error probability is computed averaging across 10^4 stochastic simulations each 80 periods long to match the length of the period 1983:1−2002:4 over which the shock processes are estimated, after discarding several periods to ascertain that the distribution did reach its long-run stationary configuration prior to the computation.

[34] The long-run stationary distribution is computed assembling 10^5 stochastic simulations at a specific time period, after discarding several periods to ascertain that the distribution did reach its long-run stationary configuration prior to the computation.

[35] The endogenous variables inherit the properties of the exogenous shock processes in a linearized model. The mark-up shock and the real-rate shock are normally distributed. Thus, also the inflation rate, the output gap, and the nominal interest rate are normally distributed if the zero bound is not considered.

[36] For the baseline, the coefficient of skewness of the nominal interest rate is 0.4 and the kurtosis is not significantly different from that of a normal distribution.

[37] For the scenario of much larger shocks, the coefficient of skewness of the nominal interest rate rises to 0.6 but the kurtosis remains not significantly different from that of a normal distribution.

	$1 \cdot \sigma_\varepsilon$	$1.5 \cdot \sigma_{\varepsilon u}$	$1.5 \cdot \sigma_{\varepsilon r}$	$1.5 \cdot \sigma_\varepsilon$
Without Zero Lower Bound:				
$E(x)$	0	0	0	0
s.d.(x)	1.2	1.7	1.2	1.7
$E(\pi)$	0	0	0	0
s.d.(π)	2.1	3.1	2.1	3.1
Fr$(\pi < 0)$	50	50	50	50
$E(i)$	3.5	3.5	3.5	3.5
s.d.(i)	2.6	3.5	3.2	4.0
Fr$(i \leq 0)$	9.2	16.0	13.7	18.8
With Zero Lower Bound:				
$E(x)$	0.0	0.0	0.0	0.0
s.d.(x)	1.2	1.6	1.3	1.8
$E(\pi)$	0.2	0.5	0.3	0.7
s.d.(π)	1.9	2.8	1.9	2.8
Fr$(\pi < 0)$	47	45	44	41
$E(i)$	3.6	3.9	3.8	4.1
s.d.(i)	2.4	3.0	2.8	3.3
Fr$(i = 0)$	3.7	8.1	6.0	8.9

Note: Results are in annualized percentage points.

Table 4: Optimal Inflation with Commitment and Larger Shocks

[Figure 2 about here]

All else equal, a higher long-run mean inflation rate allows the policymaker to support a higher long-run mean nominal interest rate and thereby protects the economy against frequent episodes of zero nominal interest rates. But inflation is costly to the economy. Thus, the optimal inflation rate resolves a long-run tradeoff between the benefit of less frequent episodes of zero nominal interest rates and the cost of higher inflation.

5.2 Optimal Inflation and Welfare Cost from the Zero Lower Bound

Table 4 shows statistics of the long-run stationary distribution as a function of the level of uncertainty about the future state of the economy. Without the zero lower bound (top panel), the long-run mean of the inflation rate, the nominal interest rate, and the output gap are independent of the level of uncertainty.[38] With the zero bound (bottom panel), however, the long-run mean inflation rate is 0.2 percent per year for the baseline, 0.3 percent per year if the real-rate shock is 50 percent more variable, and 0.5 percent per year if the mark-up shock is 50 percent more variable. The long-run mean inflation rate rises further to 0.7 percent per year if both type of shocks are 50 percent more variable.[39]

[38] The zero lower bound is the only nonlinearity in the model. Thus, the stochastic steady state coincides with the deterministic steady state if the zero bound is not considered.

[39] With the zero lower bound, the long-run mean inflation rate is less than 0.01 percent per year in a model with only a mark-up shock and 0.1 percent per year in a model with only a real-rate shock. Thus, both type of shocks are important

		$1 \cdot \sigma_\varepsilon$	$1.5 \cdot \sigma_{\varepsilon u}$	$1 \cdot \sigma_{\varepsilon r}$	$1.5 \cdot \sigma_\varepsilon$
$+$	Consumption Loss With Zero Lower Bound: μ	-0.28	-0.63	-0.29	-0.64
$-$	Consumption Loss Without Zero Lower Bound: μ_{LQ}	-0.28	-0.62	-0.28	-0.62
$=$	Additional Loss from the Zero Lower Bound: $\Delta(\mu)$	-0.00	-0.01	-0.01	-0.02

Note: Results are in annualized percentage points.

Table 5: Welfare Cost from the Zero Lower Bound with Commitment and Larger Shocks

Table 4 shows also that the frequency of zero nominal interest rates is lower if the zero bound is not considered. The optimal (long-run mean) inflation rate is not so high that the zero bound is never hit. With the zero bound, the frequency of zero nominal interest rates is 3.7 percent for the baseline, and rises to 8.9 percent if both type of shocks are 50 percent more variable.

Table 5 shows the representatives consumer's permanent consumption loss due to business cycle fluctuations. Appendix A.3 explains the computation of the permanent consumption loss.[40] The table shows the permanent consumption loss with the zero bound μ and the loss without the zero bound μ_{LQ}. Then, the additional loss due to the zero bound is $\Delta(\mu) \equiv \mu - \mu_{LQ} \leq 0$.

The welfare cost from the zero bound increases if either type of shocks is more variable. However, the welfare cost is not very large under optimal policy.[41] Even allowing for real-rate shocks and mark-up shocks 50 percent more variable than the baseline, the welfare cost from the zero bound amounts to a permanent consumption loss smaller than 0.02 percent per year.

5.3 Robustness of Results to Extreme Calibrations

Table 6 shows the results for a wide range of changes to each parameter. The parameter change can affect the results via two main channels. The parameter change modifies the 'weight' (7) the policymaker assigns to the goal of output stability relative to price stability. The parameter change also modifies the intertemporal equilibrium conditions and the 'slope' (8) of the aggregate-supply curve.

The real-rate elasticity of the output gap ($\varphi > 0$) determines the leverage of nominal-interest-rate policy on consumption. Higher elasticity implies stronger leverage and thereby lowers the frequency of zero nominal interest rates. With higher elasticity, the weight on output stability falls which makes lower variability of inflation desirable, but the slope of the aggregate-supply curve also falls which makes more variability of inflation desirable instead. Depending on which of these two opposing effects dominates, the variability of inflation can turn out either higher or lower. Thus, it is not obvious whether optimal inflation should be higher or lower. With a very high real-rate elasticity of the output gap—φ equal to 10—the optimal (long-run mean) inflation rate rises to 0.3 percent per year.

for the estimation of the optimal inflation rate even in the baseline. In addition, a model with only a real-rate shock cannot explain the variation of inflation in the data. In fact, the standard deviation of inflation explained by the real-rate shock alone is only 0.1 percent per year.

[40] The unconditional loss is computed as the mean discounted loss across $5 \cdot 10^3$ stochastic simulations each 10^3 periods long, after discarding several periods to ascertain that the distribution did reach its long-run stationary configuration prior to the computation.

[41] The welfare cost of 'suboptimally' high inflation is expected to be greater than the welfare cost under optimal policy.

Alternative Calibrations	$E(x)$	s.d.(x)	$E(\pi)$	s.d.(π)	$E(i)$	s.d.(i)	Fr$(i=0)$	$\Delta(\mu)$
Baseline	0.0	1.2	0.2	1.9	3.6	2.4	3.7	−0.00
Low elasticity of output gap ($\varphi = 1$)	0.0	0.8	0.0	1.5	3.5	3.1	11.6	−0.01
High elasticity of output gap ($\varphi = 10$)	0.0	1.2	0.3	2.0	3.8	2.3	2.9	−0.00
Extremely flexible prices ($\alpha = 0.1$)	0.0	0.2	0.1	0.3	3.6	1.6	0.0	−0.00
Very flexible prices ($\alpha = 0.3$)	0.0	0.6	0.2	0.8	3.6	1.7	0.1	−0.00
More flexible prices ($\alpha = 0.5$)	0.0	0.9	0.2	1.4	3.6	2.1	1.7	−0.00
Very sticky prices ($\alpha = 0.9$)	0.0	2.1	0.1	2.6	3.5	2.9	9.7	−0.04
Very low competition ($\theta = 3$)	0.0	0.5	0.5	2.0	4.0	2.3	1.5	−0.00
Very high competition ($\theta = 15$)	0.0	2.1	0.0	1.9	3.5	2.6	5.1	−0.02
Inelastic marginal cost ($\omega = 0.1$)	0.0	1.2	0.2	1.9	3.6	2.4	3.6	−0.00
Elastic marginal cost ($\omega = 10$)	0.0	1.2	0.2	2.0	3.6	2.4	3.7	−0.06
No inflation indexation ($\gamma = 0$)	0.0	2.0	0.0	0.9	3.5	1.6	1.3	−0.00
Less inflation indexation ($\gamma = 0.85$)	0.0	1.2	0.1	1.7	3.6	2.2	3.0	−0.00
More inflation indexation ($\gamma = 0.95$)	0.0	1.2	0.4	2.4	3.8	2.8	5.3	−0.01
Almost full indexation ($\gamma = 0.99$)	0.0	1.2	0.7	3.0	4.2	3.3	6.8	−0.01
Low steady state real rate ($r_{ss} = 2\%$)	0.0	1.2	0.6	1.8	2.5	2.1	9.2	−0.01
High steady state real rate ($r_{ss} = 5\%$)	0.0	1.1	0.1	2.0	5.0	2.5	0.4	−0.00

Notes: Results are with the zero lower bound. Results are in annualized percentage points.

Table 6: Robustness of Results to Extreme Calibrations under Commitment

The optimal inflation rate goes down for both very low and very high degrees of price stickiness ($0 < \alpha < 1$). If prices are more flexible, the variability of inflation is lower and the zero bound is reached less often, so there is less incentive for policy to support high inflation. Conversely, if prices are more sticky, the variability of inflation is higher and the zero bound is reached more often, so high inflation is more costly. The results show that for both very low and very high degrees of price stickiness—α equal to 0.1 and 0.9—the optimal inflation rate falls to 0.1 percent per year.

The price elasticity of demand substitution among differentiated goods produced by firms ($\theta > 1$) determines the degree of monopolistic competition. The less competition among firms, the stronger the incentive for policy to support high inflation, which restrains the frequency of zero nominal interest rates. With very low competition—θ equal to 3—the optimal inflation rate rises to 0.5 percent per year.

The elasticity of a firm's real marginal cost with respect to its own output level ($\omega > 0$) has little effect on the results. Less elastic marginal cost produces stronger incentives for policy to avoid zero nominal interest rates. However, such incentives are not strong enough to generate any noticeable change in the optimal inflation rate.

The policymaker's incentive to tolerate inflation variability is a function of the degree of inflation indexation ($0 \leq \gamma < 1$). The policymaker is less reluctant to tolerate inflation variability if there is more inflation indexation, because only the unanticipated component of price changes matters for social welfare. The more inflation indexation, the higher the frequency of zero nominal interest rates and the higher the optimal inflation rate. With almost full inflation indexation—γ equal to 0.99—the optimal

Lucas (2000) surveys research on the welfare cost of suboptimal inflation in a monetary economy.

inflation rate rises to 0.7 percent per year.[42]

A lower deterministic steady state real interest rate ($0 < r_{ss} < +\infty$) implies a lower deterministic steady state nominal interest rate, and thereby the zero bound is reached more often. When interest rates are lower, the policymaker has a stronger incentive to support higher inflation. With a very low deterministic steady state real interest rate—r_{ss} equal to 2 percent per year—the optimal inflation rate rises to 0.6 percent per year.

The findings overall are robust to a wide range of changes to each parameter. Although changing one parameter at a time helps understand the role of a particular dimension of the model in generating the results, there may be uncertainty along multiple dimensions of the model. As a result, changing only one parameter may understate the extent of the model misspecification.

The researcher may assess the extent of the model misspecification changing all the parameters of the model at once, each in the direction that gives rise to the worst scenario. Yet, it is not clear whether the worst scenario is towards the boundary of the joint parameter space. As table 6 shows, the worst scenario for the optimal (long-run mean) inflation rate occurs for intermediate values of the degrees of price stickiness ($0 < \alpha < 1$) rather than at the boundary.

6 Optimal Inflation With Commitment and Model Uncertainty

6.1 Long-Run Stationary Distribution

Figure 3 shows the long-run stationary distribution for the baseline in the worst-case equilibrium with extreme model uncertainty. The lowest detection error probability for which the algorithm can identify an equilibrium with the zero lower bound is 29 percent. The distribution of inflation and the output gap is almost normal and symmetric around the long-run unconditional mean. At the same time, the mean inflation rate and the mean nominal interest rate are higher if the zero bound is considered.[43]

[Figure 3 about here]

Figure 4 shows the long-run stationary distribution in the approximating equilibrium without a worst-case shock. The mean inflation rate and the mean nominal interest rate are not as high as in the worst-case equilibrium if the zero bound is considered. Intuitively, the approximating equilibrium is an 'intermediate' case between the worst-case equilibrium and the equilibrium without model uncertainty.

[Figure 4 about here]

[42] For the reasons explained in footnote 30, the scenario of full indexation is not well defined because the inflation rate would be nonstationary.

[43] For the worst-case equilibrium with extreme model uncertainty, the coefficient of skewness of the nominal interest rate is 0.6 and the kurtosis is not significantly different from that of a normal distribution.

6.2 Optimal Inflation and Welfare Cost from the Zero Lower Bound

Figure 5 shows statistics of the long-run stationary distribution of inflation as a function of the degree of model uncertainty. The detection error probability is between 29 and 50 percent if the zero bound is considered.[44] The left-hand panel shows the worst-case equilibrium and the right-hand panel shows the approximating equilibrium without a worst-case shock. The optimal long-run mean inflation rate (top panel) is zero for any degree of model uncertainty if the zero bound is not considered.[45] However, the optimal inflation rate accounting for the zero bound is increasing in a nonlinear fashion with the degree of model uncertainty. The optimal inflation rate in the worst-case equilibrium rises from 0.2 percent per year without model uncertainty to 0.9 percent per year when the detection error probability is 29 percent. Thus, the worst-case equilibrium encompasses each parameter change in table 6.

[Figure 5 about here]

Figure 5 shows also that the standard deviation of inflation (middle panel) in the worst-case equilibrium rises from 1.9 percent per year without model uncertainty to 2.9 percent per year when the detection error probability is 29 percent. At the same time, the frequency of deflation (bottom panel) in the worst-case equilibrium falls from 47 to 40 percent if there is model uncertainty. Intuitively, a higher long-run mean inflation rate protects the economy against frequent episodes of deflation if there is model uncertainty.[46]

The model provides estimates of the optimal inflation rate based on a hypothetical measure of inflation without measurement bias. As explained in section 1, however, available measures of inflation tend to be biased upward. Thus, an estimate of the measurement bias has to be added to convert the model-based estimates into actual measures of inflation.

Figure 6 compares the optimal inflation rate in the worst-case equilibrium without measurement bias (left-hand panel) to the optimal inflation rate with 0.5 percentage point per year measurement bias (right-hand panel). With measurement bias, the optimal inflation rate rises from 0.7 percent per year without model uncertainty to 1.4 percent per year when the detection error probability is 29 percent.[47]

[Figure 6 about here]

Figure 7 shows statistics of the long-run stationary distribution of the nominal interest rate as a function of the degree of model uncertainty. The optimal long-run mean nominal interest rate (top panel) accounting for the zero bound is increasing in a nonlinear fashion with the degree of model uncertainty. The optimal nominal interest rate in the worst-case equilibrium rises from 3.6 percent

[44] A detection error probability even as low as 10 percent—not shown in the figures—is feasible if the zero lower bound is not considered. Thus, models which abstract from the zero bound overstate the feasible degree of model uncertainty.

[45] See footnote 38.

[46] The long-run mean output gap is approximately zero for any degree of model uncertainty. With or without the zero lower bound, the standard deviation of the output gap in the worst-case equilibrium rises from 1.2 percent without model uncertainty to 1.6 percent when the detection error probability is 29 percent.

[47] The frequency of deflation is lower with measurement bias because the long-run stationary distribution of inflation is shifted by the estimate of the measurement bias.

per year without model uncertainty to 4.3 percent per year when the detection error probability is 29 percent. At the same time, the standard deviation of nominal interest rate (middle panel) in the worst-case equilibrium rises from 2.4 to 3.2 percent per year, and the frequency of zero nominal interest rates (bottom panel) rises from 3.7 to 7.0 percent. Intuitively, a higher long-run mean nominal interest rate protects the economy against frequent episodes of zero nominal interest rates if there is model uncertainty.

[Figure 7 about here]

Figure 8 shows the representatives consumer's permanent consumption loss due to business cycle fluctuations over the probability of a detection error. The top panel shows the worst-case equilibrium and the bottom panel shows the approximating equilibrium without a worst-case shock. Model uncertainty produces a larger permanent consumption loss. However, the welfare cost from the zero lower bound—distance between the line with circles (loss with the zero bound) and the line with squares (loss without the zero bound)—amounts to a permanent consumption loss smaller than 0.02 percent per year.

[Figure 8 about here]

6.3 Worst-Case Shocks

Without the zero lower bound, the simpler problem is standard linear-quadratic and the worst-case shocks are a linear function of the state variables. For example, the worst-case shock to the aggregate-supply curve has this linear representation: $w_{1t} = 0.34u_t + 0r_t^n + 0\pi_{t-1} + 0.17m_{1t-1} + 0m_{2t-1}$ when the detection error probability is 29 percent. The worst-case shock to the Euler equation instead is independent of the sate of the economy, or $w_{2t} = \underline{0}\cdot s_t$, for any degree of model uncertainty, because the Euler equation is irrelevant to the solution. With the zero bound, however, the problem in nonlinear and the worst-case shocks do not have a closed-form solution.

Figure 9 compares the worst-case shock to the aggregate-supply curve (left-hand panel) and the worst-case shock to the Euler equation (right-hand panel), where the latter is relevant if the zero bound is considered. The more variable the worst-case shocks (top panel), the more severe the model uncertainty. The correlation of the worst-case shocks to either the mark-up shock (middle panel) or the real-rate shock (bottom panel) is non negative when the zero bound is considered, which shows that the worst-case shocks are 'piled onto' the exogenous shock processes.

[Figure 9 about here]

The left-hand panel of figure 9 shows that the worst-case shock to the aggregate-supply curve piles onto the mark-up shock, but not the real-rate shock. Thus, model uncertainty in the aggregate-supply curve does not propagate to the Euler equation. At the same time, the right-hand panel shows that the worst-case shock to the Euler equation piles onto the mark-up shock as well as the real-rate shock if the zero bound is considered. Thus, model uncertainty in the Euler equation propagates to the aggregate-supply curve.

7 Optimal Inflation with Commitment to a Taylor Rule

Thus far, the analysis relies on the policymaker's commitment to an equilibrium path for the economy that achieves time-zero optimal policy and maximizes welfare for the consumer. But, to investigate the effects of policy commitment on optimal inflation, in this section the policymaker commits merely to a Taylor rule rather than the equilibrium path.[48]

7.1 Commitment to a Taylor Rule

Instead of committing to an equilibrium path, the policymaker sets the nominal interest rate as a function of the deviation of inflation from an inflation target π^* and the deviation of the output gap from an output gap target x^*:

$$i_t = \max\left[0, (1-\phi_i)\,\bar{\imath} + \phi_i i_{t-1} + \phi_\pi (\pi_t - \pi^*) + \phi_x (x_t - x^*)\right] \tag{17}$$

where $\bar{\imath} = r_{ss} + \pi^*$ is the equilibrium level of the nominal interest rate when inflation and the output gap are at target, and $\phi_\pi, \phi_x \geq 0$ are the policy response coefficients on inflation and output gap deviations from target.[49] In addition, $\phi_i \geq 0$ is the degree of 'policy inertia' or the weight on last period's nominal interest rate level. The rule explicitly takes into account that the policymaker cannot achieve negative nominal interest rates.

Woodford (2003a) shows that when the policymaker uses a rule similar to (17), but can set nominal interest rates to negative values, equilibrium is determinate if and only if the policy response coefficients satisfy $\phi_\pi > -(1-\beta)\,\kappa^{-1}\phi_x + 1 - \phi_i$. Thus, the equilibrium is determinate regardless of how small the policy response to changes in inflation ($\phi_\pi > 0$) when the rule is 'superinertial' or $\phi_i \geq 1$. Intuitively, if the rule is superinertial, a sustained fall in inflation results in an even greater sustained fall in the nominal interest rate.

Once the zero lower bound on nominal interest rates is explicitly taken into account, however, the policymaker cannot respond too aggressively to changes in inflation to ensure determinacy of equilibrium in the Taylor rule (17). Intuitively, the policymaker cannot stabilize the economy if nominal interest rates are excessively variable and thereby the zero lower bound is encountered too often. Thus, the policy response coefficients have an upper bound, $\bar{\phi}_j \subset (0, +\infty)$ for $j = \pi, x$, beyond which determinacy of equilibrium cannot be not ensured.

[48] The model is the same as in section 2, but the Taylor rule (17) replaces the objective function (1). Thus, the policy problem is now given by equations (2)-(6) for $w_{1t} \equiv w_{2t} \equiv 0$, and (17). Worst-case scenarios of model uncertainty are not considered, because the robust control approach of Hansen and Sargent (2008) is not applicable unless the policymaker directly uses an objective function.

[49] The output gap target x^* is the deterministic steady state value of the output gap consistent with the inflation target π^* in the aggregate-supply curve (2). Thus, $x^* = (1-\gamma)(1-\beta)\,\kappa^{-1}\pi^*$.

7.2 Equilibrium

Equations (2), (3), and (17) form a nonlinear system of three equations with three unknowns, which must be satisfied by policy in equilibrium. Solving the system gives a three-dimensional nonlinear equilibrium response function $y(s_t) \equiv (\pi_t, x_t, i_t \geq 0) \subset R^3$ over a four-dimensional state space $s_t \equiv (u_t, r_t^n, \pi_{t-1}, i_{t-1} \geq 0) \subset R^4$. However, i_{t-1} is not a state variable if $\phi_i = 0$, because the nominal interest rate level in period $t-1$ is irrelevant for the conduct of policy in period t when there is no policy inertia.[50] Then, the following equilibrium definition is proposed.

Definition 2 (SREE-TR) *Assume $\sigma_{\varepsilon j} \geq 0$ for $j = u, r$. A 'stochastic rational expectations equilibrium' of the policy problem (2)-(6) with commitment to the Taylor rule (17) is a nonlinear response function $y(s_t)$, over the state s_t, with a law of motion, such that the nonlinear system of equilibrium conditions (2), (3), and (17) is satisfied.*[51]

7.3 Optimal Inflation

As originally proposed by Taylor (1993), ϕ_π is 1.5 and ϕ_x is 0.5, which makes the Taylor rule an approximate description of monetary policy under Greenspan's chairmanship of the U.S. Federal Reserve. In addition, π^* is the lowest inflation target which maximizes welfare for the consumer.[52]

Table 7 shows the results as a function of the degree of policy inertia.[53] Without policy inertia ($\phi_i = 0$), the long-run mean inflation rate is as high as 8 percent per year. However, the long-run mean inflation rate is lower when policy is inertial. The long-run mean inflation rate is even lower when policy is superinertial. Intuitively, policy inertia mimics optimal policy behavior because the policymaker can steer private-sector expectations of future policy more effectively when current policy actions depend on the past state of the economy. If $\phi_i \geq 6$, the Taylor rule can practically implement the optimal long-run mean inflation rate achieved under commitment to an equilibrium path.

Table 7 shows also that the consumer's permanent consumption loss due to business cycle fluctuations is smaller when policy is more inertial.[54] If the policy inertia rises beyond a certain threshold, however, the permanent consumption loss is bigger because policy becomes somewhat less flexible.[55] Thus, there is an optimal degree of policy inertia. Moreover, the optimal (long-run) mean inflation rate under the Taylor rule may not be as low as the optimal inflation rate achieved under commitment to an equilibrium path. In fact, the consumer's welfare is maximized for $\phi_i = 5$ and the optimal inflation rate is 0.5 percent per year, which is a little higher than the optimal inflation rate under commitment to an

[50] As explained for the equilibrium of the optimal policy problem in section 3, the equilibrium response function has an associated expectations function, which simplifies when agents have perfect foresight.

[51] The numerical procedure in appendix A.1 is used to find the nonlinear equilibrium response function.

[52] More specifically, π^* is also the lowest inflation target for which the numerical procedure can identify an equilibrium.

[53] Since the frequency of zero nominal interest rates is approximately zero for any degree of policy inertia in the table, the response function is approximately linear in equilibrium and the long-run mean inflation rate is approximately equal to the inflation target.

[54] The permanent consumption loss is evaluated on the welfare-theoretic objective function (1).

[55] The autocorrelation of the nominal interest rate is between 0.83 and 0.86 for the range of policy inertia in the table.

Policy Inertia: ϕ_i	$E(x)$	s.d.(x)	$E(\pi)$	s.d.(π)	$E(i)$	s.d.(i)	Fr($i=0$)	μ	$\Delta(\mu)$
0	0.1	0.9	8.0	1.9	11.5	2.6	0.0	−0.46	−0.00
1	0.0	1.0	4.5	1.4	8.0	1.8	0.0	−0.32	−0.00
2	0.0	1.2	3.0	1.3	6.5	1.4	0.0	−0.29	−0.00
3	0.0	1.5	2.0	1.3	5.5	1.1	0.0	−0.28	−0.00
4	0.0	1.6	1.0	1.3	4.5	1.0	0.0	−0.28	−0.00
5	0.0	1.7	0.5	1.3	4.0	0.8	0.0	−0.28	−0.00
6	0.0	1.8	0.2	1.4	3.7	0.7	0.0	−0.29	−0.00
7	0.0	1.9	0.2	1.4	3.7	0.7	0.0	−0.29	−0.00
8	0.0	2.0	0.2	1.4	3.7	0.6	0.0	−0.29	−0.00

Notes: The policymaker commits to a Taylor rule subject to a zero lower bound. The optimal inflation rate is the lowest long-run inflation target which maximizes social welfare. Results are in annualized percentage points.

Table 7: Optimal Inflation with Commitment to a Taylor Rule

equilibrium path or 0.2 percent per year.[56]

8 Conclusions

Previous studies show that the frequency of zero nominal interest rates falls quickly as the central bank's long-run inflation goal rises above zero percent. Previous studies, however, do not provide a direct estimate of the optimal (long-run mean) inflation rate that maximizes the public's economic well-being, which is essential information for formulating a long-run inflation goal.

This paper provides direct estimates of the optimal inflation rate in a small New-Keynesian model subject to the zero lower bound. In addition, the paper shows the robustness of the estimates to worst-case scenarios of model uncertainty. The optimal inflation rate is estimated between 0.7 percent per year (no model uncertainty) and 1.4 percent per year (extreme model uncertainty) when measured using the PCE price index.

The paper shows also the robustness of these estimates when the policymaker commits merely to a Taylor rule, rather than a completely specified policy plan. By using a superinertial Taylor rule with a sufficiently high degree of policy inertia, the policymaker can practically implement the optimal inflation rate that maximizes the public's economic well-being. The optimal inflation rate, however, is more than 14 percent per year when the policymaker makes no commitment about future policies or re-optimizes every period.

A number of caveats must be kept in mind. First, the model focuses on the effects of the zero lower bound and ignores other factors that may influence policymakers when formulating the long-run inflation goal, such as downward wage rigidity or debt-deflation. Second, measurement bias in the various price indexes creates some uncertainty around the estimates. Finally, the estimates are derived from a very

[56] The permanent consumption loss is approximately the same as under commitment to an equilibrium path or −0.28 percent per year (table 5).

simple model that abstracts from many real-world features. Thus, further research is needed to confirm or refine the estimates in models that incorporate a more complete description of the economy.

A Appendix

A.1 Numerical Procedure

The state space s is discretized into a set N of interpolation nodes $\{s_n|n = 1, ..., N\}$ where $s_n \in s$. The response function $\hat{y}(s)$ is evaluated at intermediate values of the discretization grid with multilinear interpolation. Since the shock innovations ε are normally distributed, the expectations function $\hat{E}\hat{y}_{+1}$ is evaluated accurately and efficiently with an M-node Gaussian-Hermite quadrature scheme $\{\varepsilon_m|m = 1, ..., M\}$ where $\varepsilon_m \in \varepsilon$, as explained in chapter 7 of Judd (1998).[57] Quadrature-based integration is accurate if the integrands to be evaluated are smooth. The integrands are smooth because the underlying distributions of inflation and the output gap are smooth, which figures 1 to 4 show.

The fixed-point of the nonlinear system is found with an iterative update rule

$$\hat{y}^{k+1} \leftarrow \hat{y}^k + \iota^k \left(\hat{y}^{k+1} - \hat{y}^k\right), \text{ from step } k \text{ to } k+1 \tag{18}$$

where $\iota^k \in (0, 1]$ is the step size to guarantees algorithm stability, as explained in chapter 4 of Bertsekas (1999).

The **Algorithm** proceeds as follows:

Step 1: Assign the interpolation nodes with an efficient, sparse-grid method. Guess an initial value $\hat{y}^0$ for the response function.

Step 2: Update the state, evaluate the expectations function, and apply the iterative update rule (18) to derive a new guess $\hat{y}^{+1}$.

Step 3: Stop if $\max_{n=1,...,N} \left\|\hat{y}^{k+1} - \hat{y}^k\right\| < \tau$, where $\tau > 0$ is the convergence tolerance level. Otherwise repeat step 2.

The convergence tolerance level is set to the square root of machine precision $\tau = 1.49 \cdot 10^{-8}$. The accuracy of the solution is checked computing the Euler equation residuals at an arbitrary set R of residual interpolation nodes $\{s_r|r = 1, ..., R\}$ where $s_r \in s$, as explained by Santos (2000). The Euler equation residuals are verified at the interpolation nodes and over a finer grid to assure that the approximation error does not affect the results.

To cope with the curse of dimensionality, the procedure employs a sparse grid which assigns more weight to the regions of the state space where the occasionally-binding constraint is active. The support for the exogenous shocks covers ± 4 unconditional standard deviations, which is large enough to drive the nominal interest rate to zero. The support for the endogenous state variables is large enough to

[57] A quadrature scheme is not necessary if agents have perfect foresight ($\sigma_\varepsilon \to 0$).

avoid erroneous extrapolation. To achieve with a sparse grid an acceptable degree of approximation for the baseline requires $N \approx 3.6 \cdot 10^4$ interpolation nodes and $M \approx 45$ quadrature nodes, as opposed to a linearly spaced grid which would require $N > 10^6$ interpolation nodes.

To achieve greater efficiency, the procedure employs an approximation refinement method. The initial guess is set to the linearized solution around the deterministic steady state on a coarse grid $N^0 \times M^0$. The nonlinear solution obtained for the coarse grid is then interpolated over a finer grid and used as a new guess to resolve. The degree of approximation is progressively increased towards the final set of nodes $\{N^0 \times M^0 < N^1 \times M^1 < \ldots < N \times M\}$. Experimentation with alternative initial guesses did not lead to differences in the results.

A.2 Detection Error Probability

As explained in chapter 9 of Hansen and Sargent (2008), the researcher should simulate the model for a small sample when computing detection error probabilities. Model misspecification is easy to detect in a long sample.[58]

The researcher can estimate the log-likelihood ratio that the data is generated by the approximating model without a worst-case shock

$$r_A = \frac{1}{T} \sum_{t=0}^{T-1} \left[\frac{1}{2} w'_{At} w_{At} - w'_{At} \varepsilon_{At} \right]$$

where w_{At} is a vector of worst-case shocks distorting the dynamics of the exogenous shock processes. ε_{At} is a vector of normally distributed innovations, independent across time and cross-sectionally. The paths for w_{At} are obtained from stochastic simulations with undistorted exogenous shock processes.

The researcher can also estimate the log-likelihood ratio that the data is generated by the approximating model with a worst-case shock

$$r_B = \frac{1}{T} \sum_{t=0}^{T-1} \left[\frac{1}{2} w'_{Bt} w_{Bt} + w'_{Bt} \varepsilon_{Bt} \right]$$

where w_{Bt} is a vector of worst-case shocks distorting the dynamics of the exogenous shock processes. ε_{Bt} is a vector of normally distributed innovations, independent across time and cross-sectionally. The paths for w_{Bt} are obtained from stochastic simulations with distorted exogenous shock processes.

Assigning equal prior weights to the approximating model with or without a worst-case shock, the overall detection error probability is

$$p(\Theta) = \frac{1}{2}(p_A + p_B)$$

where $p_j = \text{Freq}(r_j \leq 0)$ for $j = A, B$.

[58] The detection error probability is 50 percent in the limit for $T \to +\infty$.

A.3 Permanent Consumption Loss

The expected life-time utility of the representative consumer, as shown in chapter 6 of Woodford (2003a), is validly approximated by

$$\hat{E}_0 \sum_{t=0}^{\infty} \beta^t U_t = \frac{U_c \overline{C}}{2} \frac{\alpha\theta(1+\omega\theta)}{(1-\alpha)(1-\alpha\beta)} \hat{L} \tag{19}$$

where $\overline{C}$ is deterministic steady state consumption, $U_c > 0$ is deterministic steady state marginal utility of consumption, and

$$\hat{L} \equiv -\hat{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(\pi_t - \gamma\pi_{t-1})^2 + \lambda x_t^2 - \Theta\left(w_{1t}^2 + w_{2t}^2\right) \right] \leq 0$$

is the welfare-theoretic objective function (1) which the policymaker maximizes.

At the same time, the utility loss from a permanent consumption loss $\mu \leq 0$ is

$$\hat{E}_0 \sum_{t=0}^{\infty} \beta^t U_c \overline{C} \mu = \frac{1}{1-\beta} U_c \overline{C} \mu \tag{20}$$

Equating the right-hand sides of (19) and (20), the permanent consumption loss is

$$\mu = \frac{1-\beta}{2} \frac{\alpha\theta(1+\omega\theta)}{(1-\alpha)(1-\alpha\beta)} \hat{L}$$

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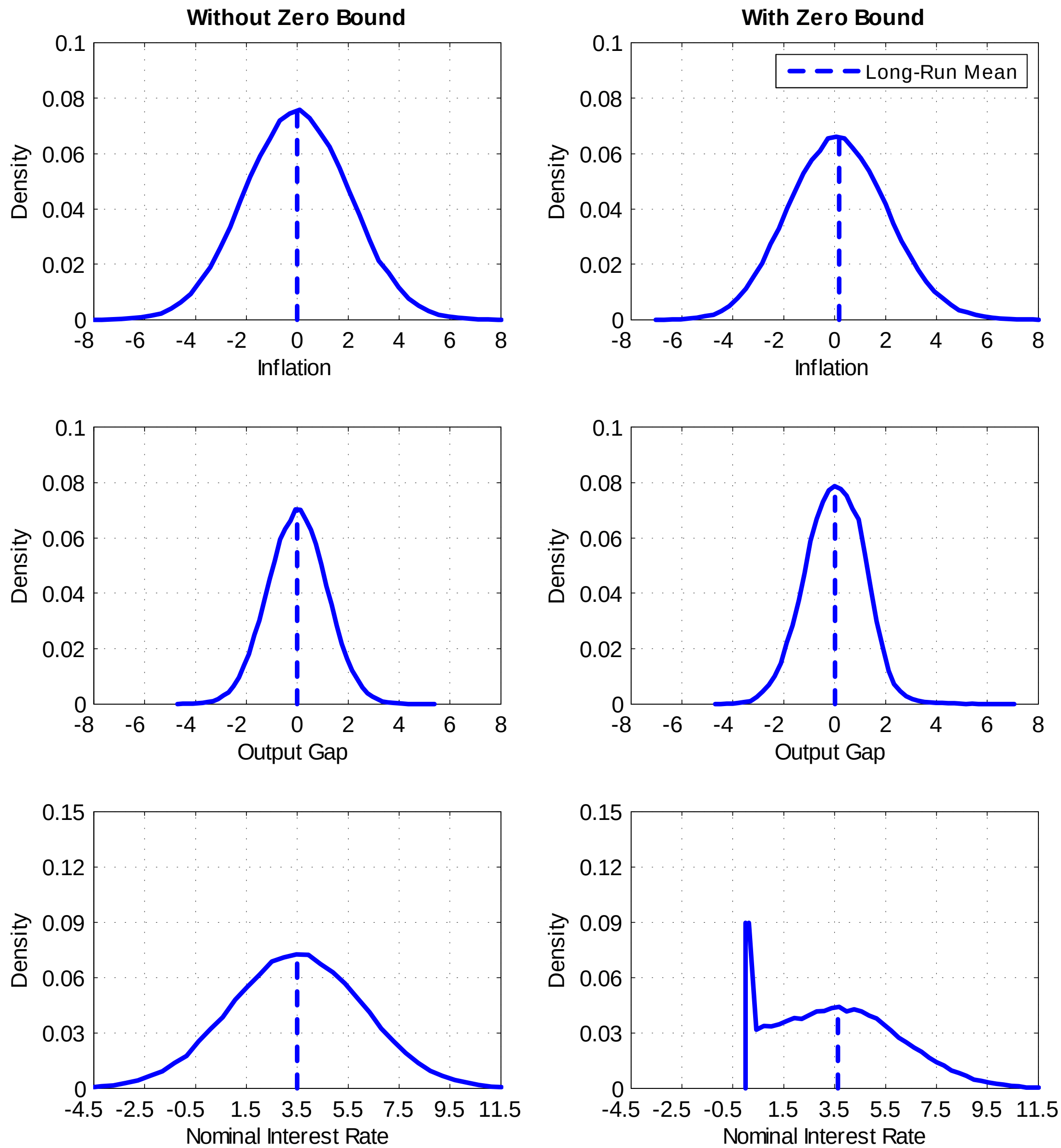

Figure 1: Long-Run Distribution for Baseline

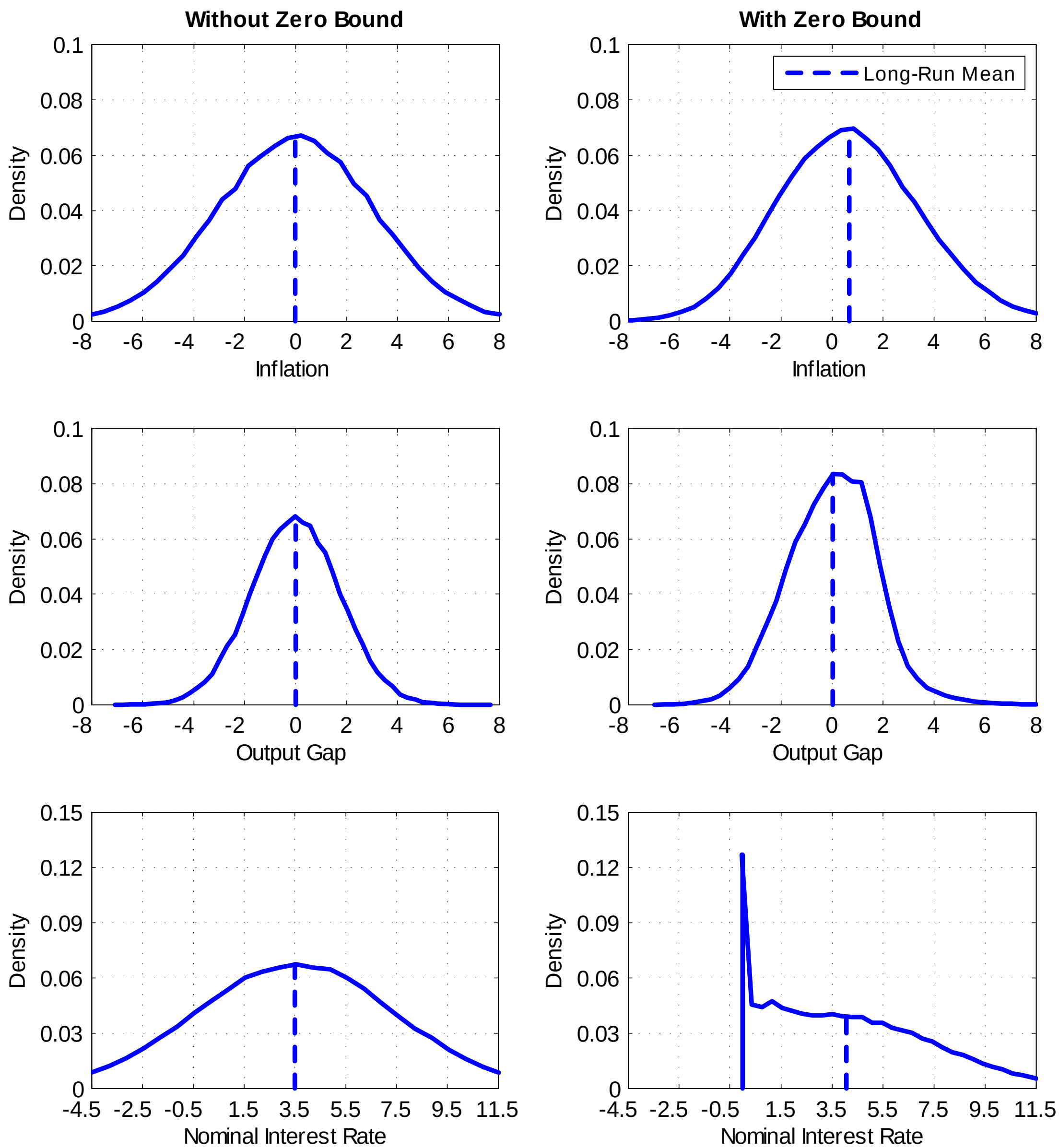

Figure 2: Long-Run Distribution with Larger Shocks

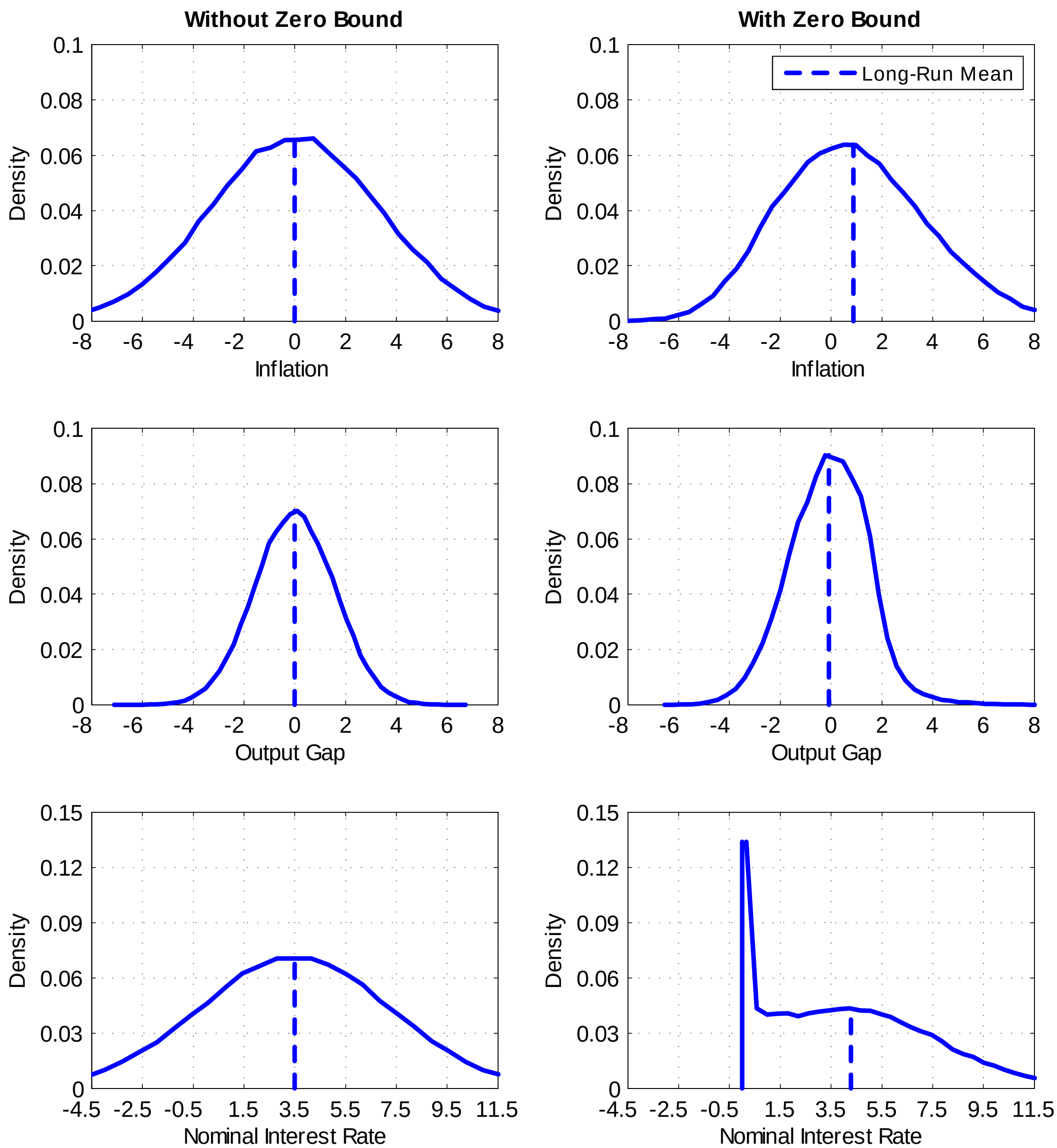

Figure 3: Long-Run Distribution with Extreme Model Uncertainty: Worst-Case Equilibrium

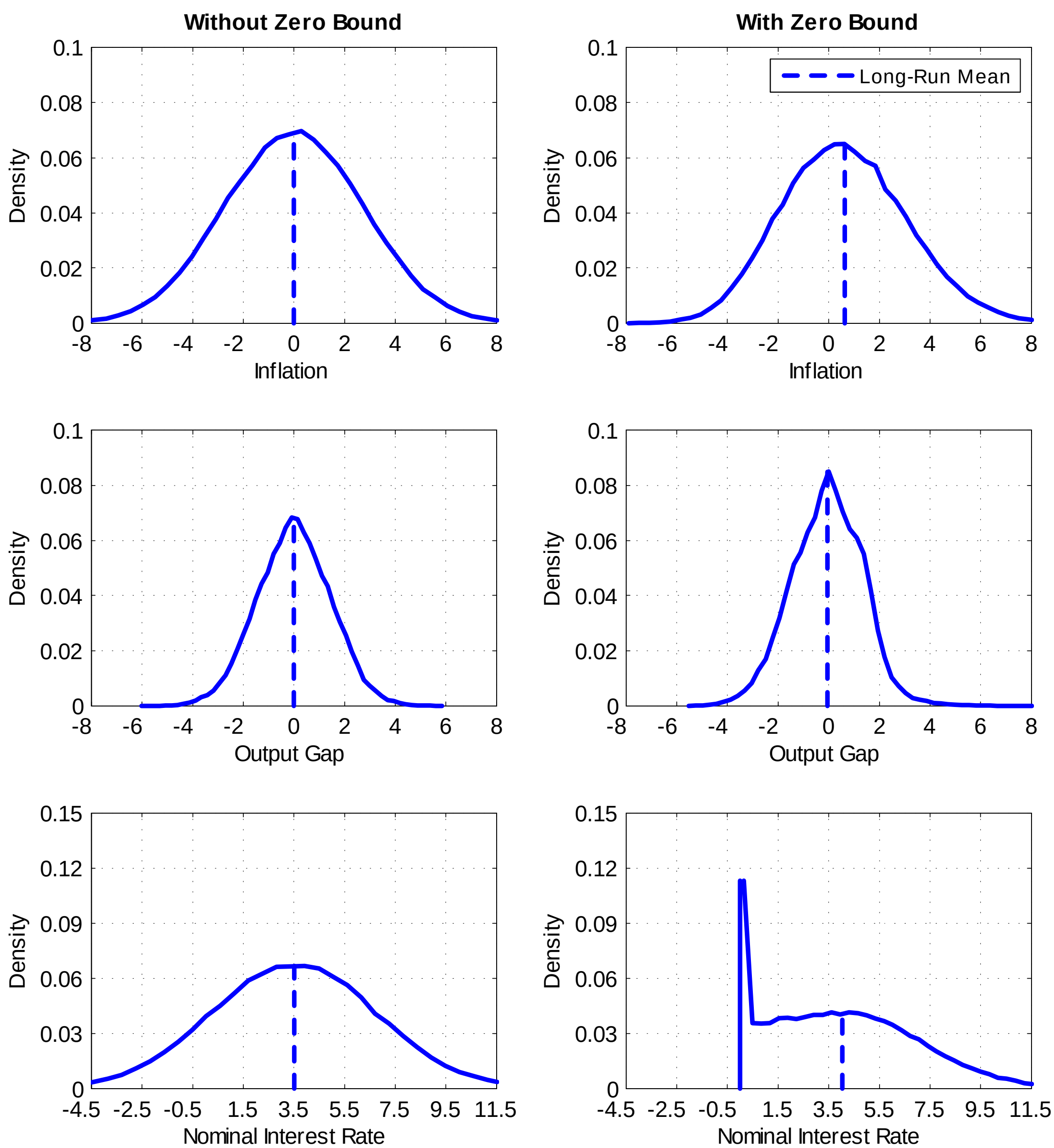

Figure 4: Long-Run Distribution with Extreme Model Uncertainty: Approximating Equilibrium

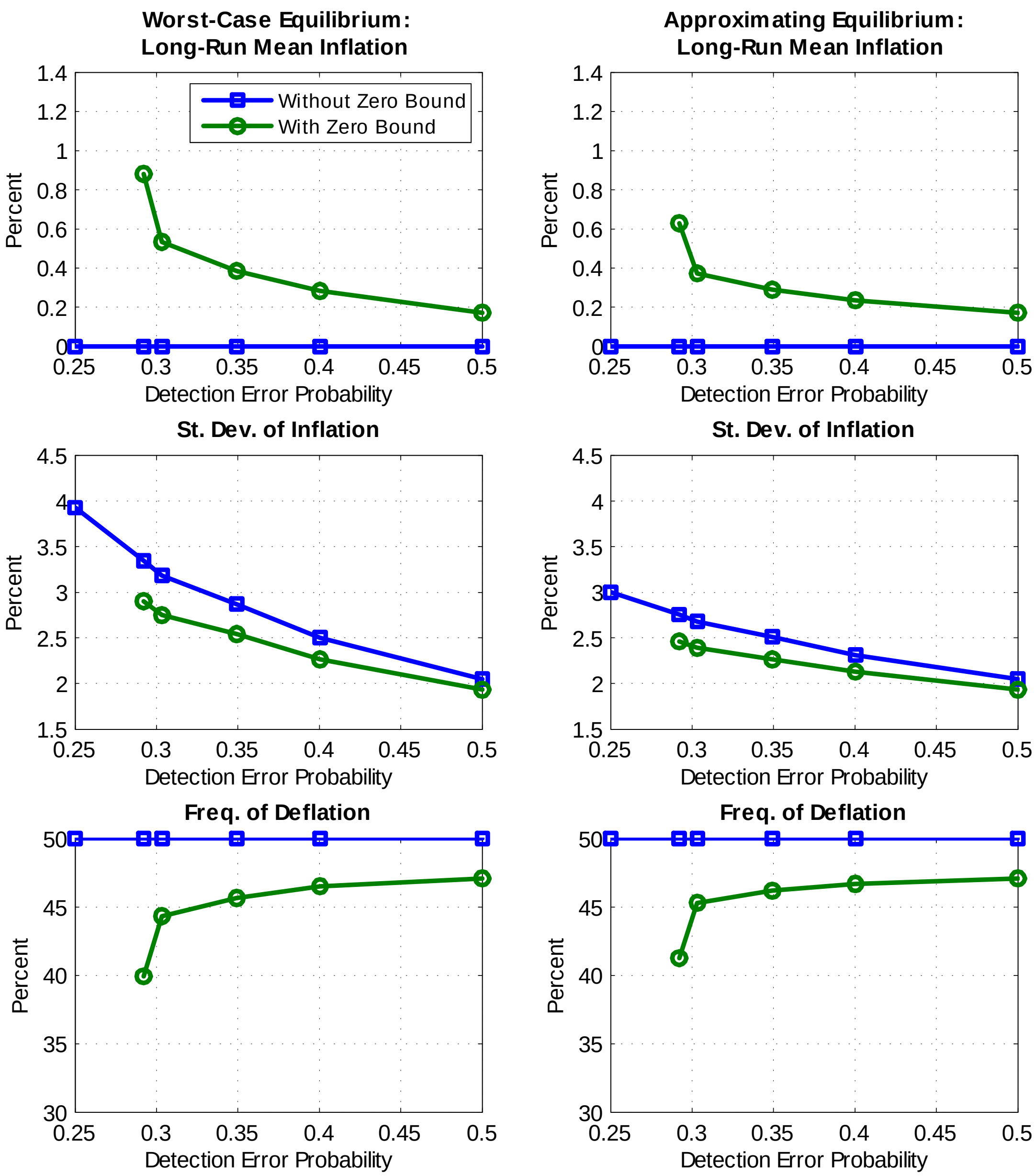

Figure 5: Optimal Inflation and Model Uncertainty

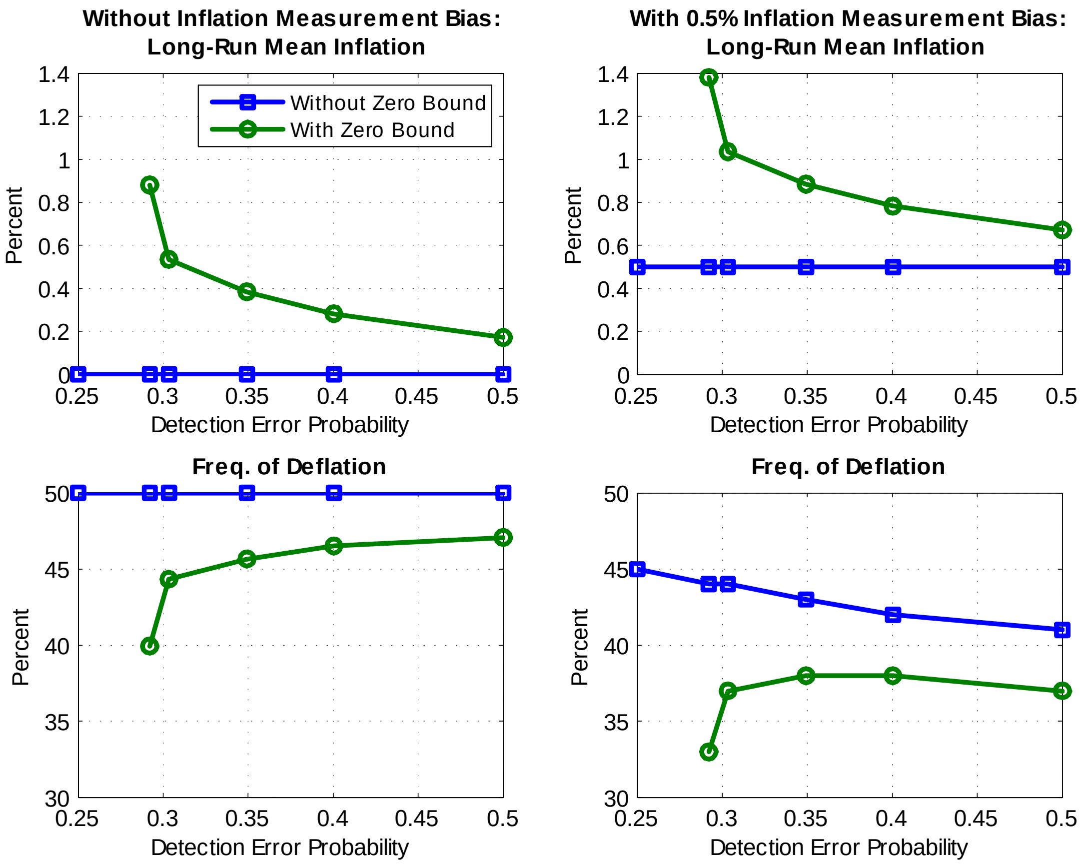

Figure 6: Optimal Inflation, Measurement Bias and Model Uncertainty: Worst-Case Equilibrium

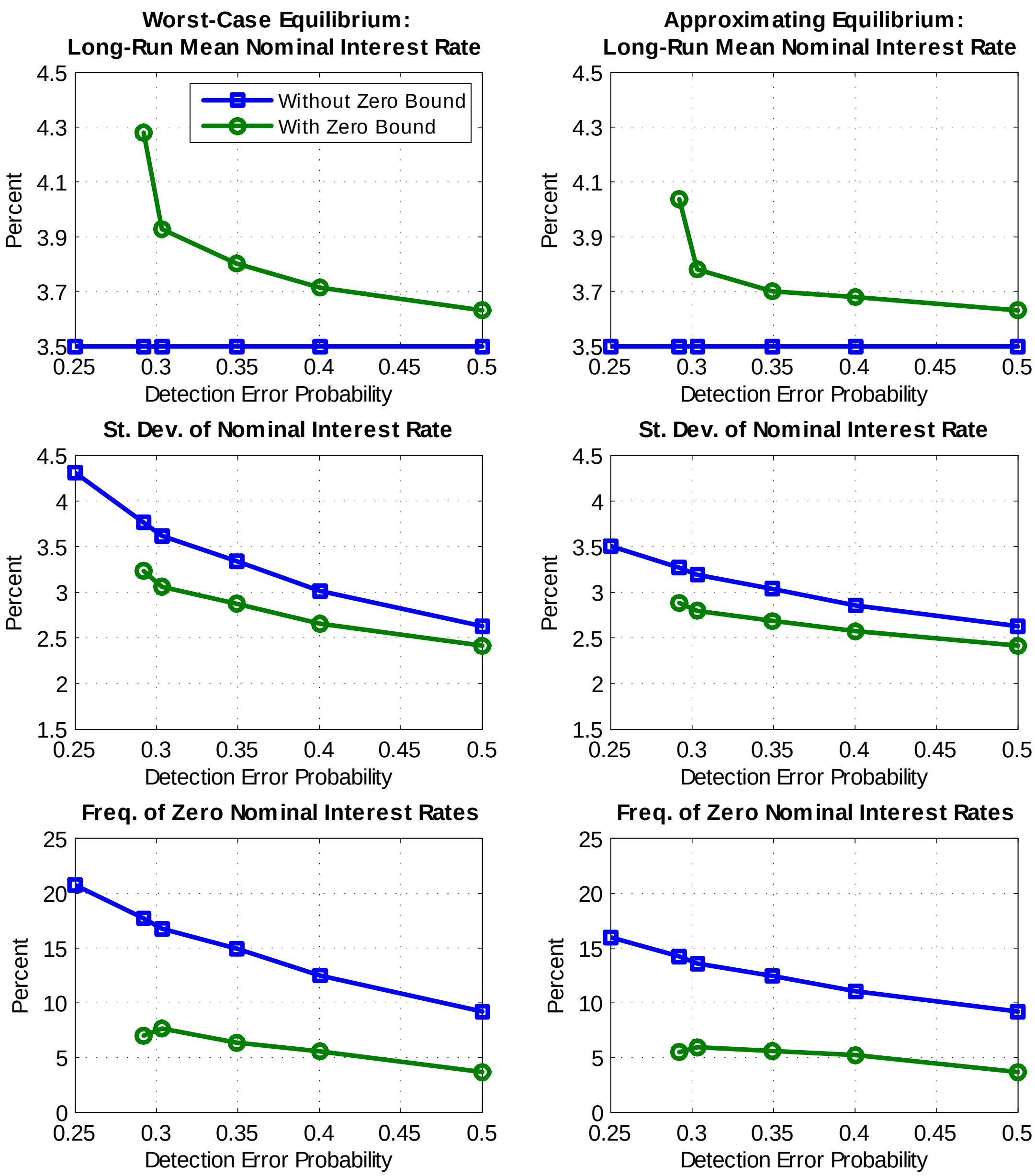

Figure 7: Optimal Nominal Interest Rate and Model Uncertainty

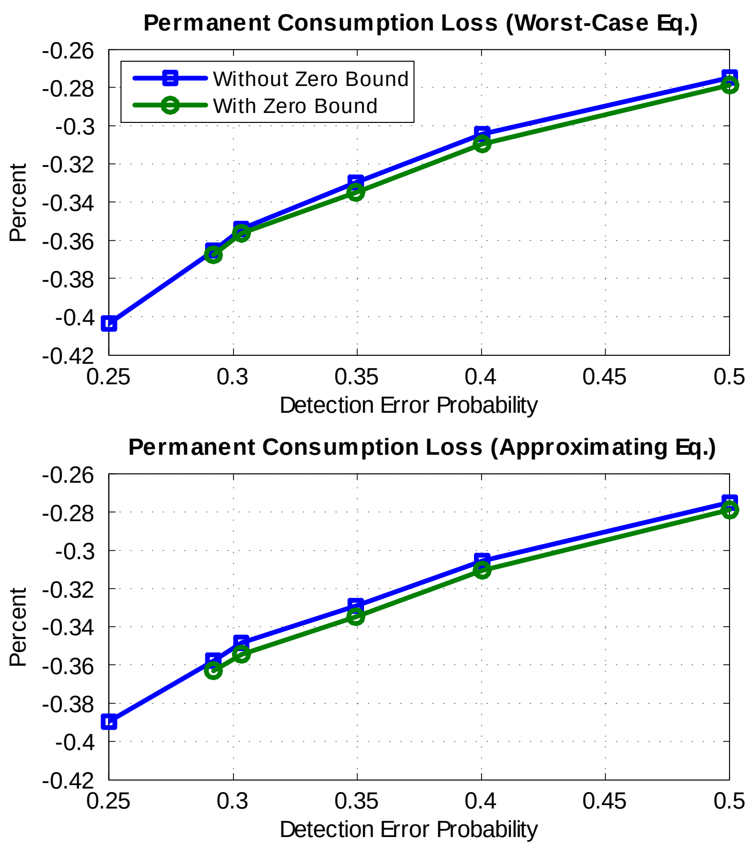

Figure 8: Welfare Cost from the Zero Lower Bound and Model Uncertainty

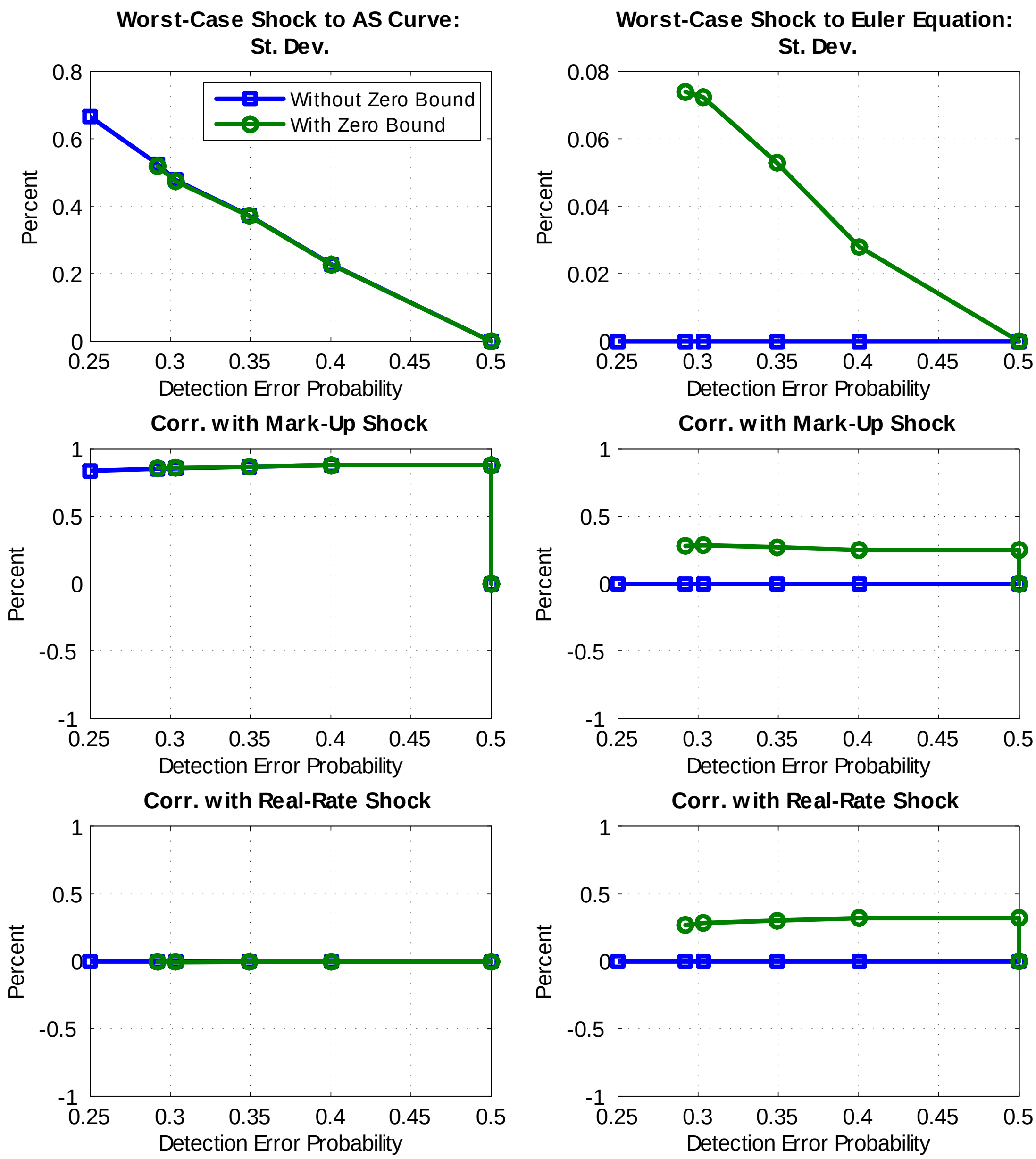

Figure 9: Worst-Case Shocks and Model Uncertainty